Hamiltonian singularities in the STIRAP process

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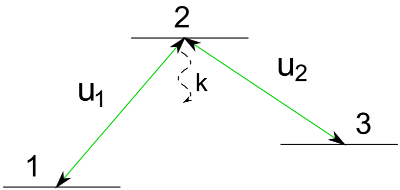
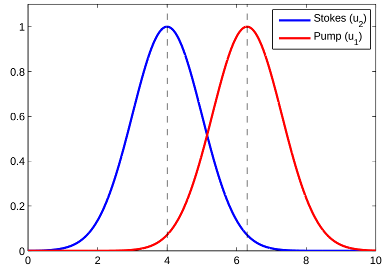
Outline

Stimulated Raman Adiabatic Passage





- A counterintuitive sequence.
- Avoids dissipation of state $|2\rangle$.

About STIRAP

- Dynamics :

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{pmatrix} = \begin{pmatrix} -u_1 x_2 \\ u_1 x_1 - k x_2 + u_2 x_3 \\ -u_2 x_3 \end{pmatrix}$$

- Dark state :

$$|\psi\rangle = \begin{pmatrix} \cos\phi \\ 0 \\ -e^{i\Phi_r}\sin\phi \end{pmatrix} \quad \text{with} \quad \tan\phi = \frac{u_1}{u_2}, \quad \Phi_r = \Phi_1 - \Phi_2$$

- An adiabatic process $\rightarrow$ infinitely long dynamics.
- A robust process.
- Optimal ? for which cost ?

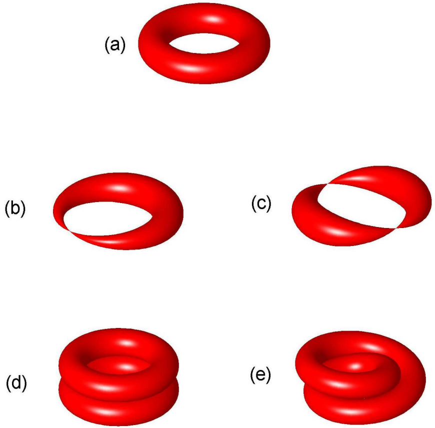
Integrable hamiltonian systems

- K is a constant of motion $\Leftrightarrow \frac{dK}{dt} = 0$.
- Integrable $\Leftrightarrow$ As many constants of motion as degrees of freedom.
- Action-Angle variables : $(p_k, q_k) \to (I_k, \theta_k)$

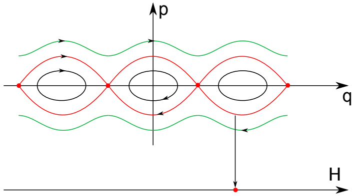
$$\left\{ \begin{array}{l} \dot{q} = \frac{\partial H}{\partial p} \\ \dot{p} = -\frac{\partial H}{\partial q} \end{array} \right. \longrightarrow \left\{ \begin{array}{l} \dot{\theta}_k = \omega(I_k) \\ \dot{I}_k = 0 \end{array} \right.$$

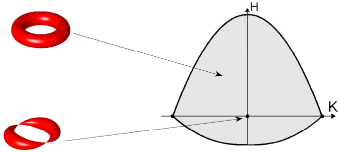
- Arnold-Liouville Theorem : Action-Angle variables exist locally.
- I_k constants $\Leftrightarrow$ a torus in the phase space.

Singular Tori



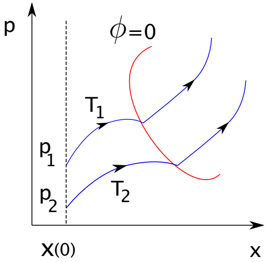
Energy-Momentum Diagram





- (H, K) space $\Leftrightarrow$ Energy-Momentum diagram.
- One value of (H, K) $\Leftrightarrow$ One torus in the phase space
- Singular tori $\Leftrightarrow$ Trajectories with infinite periods.

Pontryagin Maximum Principle



Dynamical system with a cost :

- $\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, u), \quad C = \int_0^T f_0(\mathbf{x}, u)$

Pseudo-Hamiltonian with co-state p :

- $\mathcal{H} = \mathbf{p} \cdot \mathbf{f}(\mathbf{x}, u) + p_0 f_0(\mathbf{x}, u)$

Maximization condition :

- $H = \max_{u \in U} \mathcal{H}(\mathbf{x}, \mathbf{p}, u)$

Hamiltonian dynamical system :

- $$\dot{\mathbf{x}} = \frac{\partial \mathcal{H}}{\partial \mathbf{p}}$$
  $$\dot{\mathbf{p}} = -\frac{\partial \mathcal{H}}{\partial \mathbf{x}}$$

Energy minimum cost

- Cost :

$$C = \int_0^T [u_1^2(t) + u_2^2(t)]dt$$

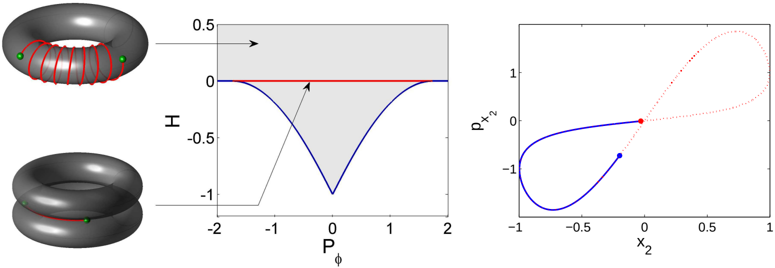
- Pseudo Hamiltonian :

$$H = -krp_r \cos^2\theta + k\cos\theta\sin\theta p_\theta + \frac{1}{2}p_\theta^2 + \frac{1}{2}\cot^2\theta {p_\phi}^2$$

with :

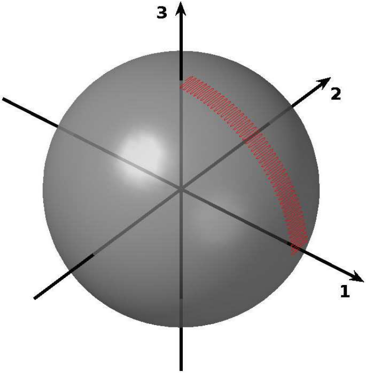
$$\begin{cases} x_1 = r\sin\theta\cos\phi \\ x_2 = r\cos\theta \\ x_3 = r\sin\theta\sin\phi \end{cases}$$

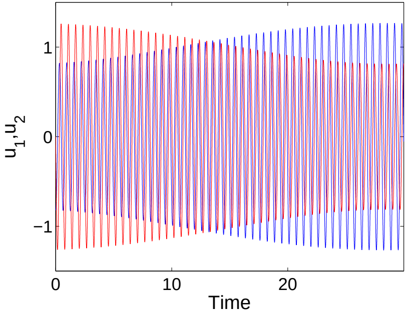
Phase space structure



- STIRAP would be on a bitorus.
- Not allowed by the dynamics.

Example of energy minimum controls





- Far from a STIRAP process.
- Avoids most of dissipation of state $|2\rangle$.

Stirap cost

- Cost :

$$C = \int_0^T \dot{\theta}^2$$

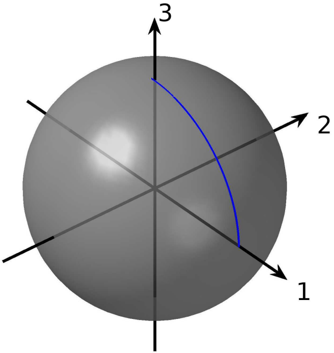
- Maximized Hamiltonian :

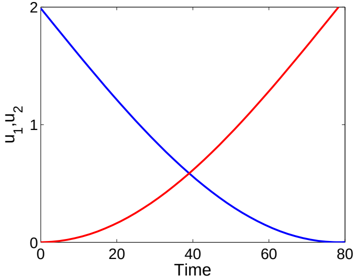
$$H = -krp_r \cos^2\theta + \frac{1}{2}p_\theta^2 - v_2 \cot\theta p_\phi$$

where

$$v_2 = -\frac{2krp_r \sin^3\theta \cos\theta}{p_\phi} \quad \text{and} \quad \theta = \frac{\pi}{2} + \epsilon$$

Example of STIRAP-like trajectory





- Counterintuitive sequence
- $\lim_{\theta \to \frac{\pi}{2}}$: $\tan \phi = \frac{u_1}{u_2}$

Summary

- STIRAP-like trajectories can be obtain with the PMP.
- It minimizes oscillations around the dark state.
- It corresponds to a singular torus of the integrable Hamiltonian system.

Perspectives

- Link between robustness and singular torus ?
- The space structure depends on the cost ?