Robust quantum control:
From adiabatic to ultrafast processes

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Transfer to a target state with
- **high fidelity** (QIP: typically error of $10^{-4}$)
- **robustness** (inhomogeneity, fluctuations, not well-known system,...)
- **speedness / low energy**

- **Rabi method**: fast, low cost but not robust
- **Adiabatic passage (RAP)**: robust, but slow / high energy, and imperfect transfer
  - Make it fast and efficient: parallel adiabatic passage
- **Composite pulses** (static phase): simple, robust, exact, but slow
  - Make it fast using time-dependent phases and tracking
- **Optimal control**: fast (optimal), but robustness complicated to treat
  (discretization and many parameters)
Unified formalism to treat Exchanges of photons between Atoms / Laser / cavity

Strong-field dressed Hamiltonian (Floquet Hamiltonian)

\[
K = H_m - \mu \left[ E_C(a + a^\dagger) + E_\ell \cos \theta \right] + \hbar \omega_C(aa^\dagger + 1/2) - i\hbar \omega_\ell \frac{\partial}{\partial \theta}
\]

Interaction

Photons

Classical fields:
\[ \theta \leftrightarrow \omega t \]

Dressed states:
\[ |m; n, \ell \rangle \]

Adiabatic theorem:
the dynamics follows the instantaneous Floquet state which is continuously connected to the initial one.

\[ t = 0: |1; 0, 0 \rangle \rightarrow |m; n, \ell \rangle \]

Robustness

Field parameters?
Optimal chirped adiabatic passage: Level lines - Parallel adiabatic passage

\[ H^{[\Omega(t), \Delta(t)]} = \frac{\hbar}{2} \begin{bmatrix} 0 & \Omega(t) \\ \Omega(t) & 2\Delta(t) \end{bmatrix} \]

Level lines = Circles

Optimal chirped adiabatic passage: Level lines - Parallel adiabatic passage

\[
H^{[\Omega(t), \Delta(t)]} = \frac{\hbar}{2} \begin{bmatrix}
0 & \Omega(t) \\
\Omega(t) & 2\Delta(t)
\end{bmatrix}
\]

Level lines = Circles

Note new toolbar buttons: data brushing & linked plots  Play video
Parallel adiabatic passage: Robustness

Ultrahigh fidelity and robustness
Usually (RAP): \( P_{\text{infidelity}} \sim \exp(- C T) \)

Parallel superadiabatic passage: \( P_{\text{infidelity}} < \exp(- C T) \)

Ex: \( \Omega(t) = \Omega_0 \sin[\theta(t)], \Delta(t) = \Omega_0 \cos[\theta(t)] \)
\( \theta(t) = \pi/2[1+\text{erf}(t/T)] \)

Parallel: \( \Omega^2(t) + \Delta^2(t) = (\Omega_0)^2 \)

\( P_{\text{infidelity}} = \exp[- C_1 T (\log(C_2 T))^{1/2}] \)
Parallel adiabatic passage: Robustness

Fluctuation of pulse area

Transitionless parallel adiabatic passage does not improve

Exponentially correlated noise of the instantaneous frequency

\[
\langle \xi(t) \rangle = 0 \\
\langle \xi(t)\xi(t') \rangle = D\Gamma \exp(-\Gamma|t-t'|)
\]
Implementation of parallel adiabatic passage with spatial light modulators

Phase spectral shaping $\phi(\omega)$ / amplitude spectral shaping $A(\omega)$

$$\mathcal{E}(t)e^{i\phi(t)} = \frac{1}{2\pi} \int_{-\infty}^{+\infty} d\omega A(\omega)e^{i(\omega t + \phi(\omega))}, \quad \Delta(t) = \phi(t) - \omega_0$$
Adiabatic control of multiphoton processes (dynamical Stark shifts)

\[
H[\Omega(t), \Delta(t)] = \begin{bmatrix}
0 & \Omega(t)/2 \\
\Omega(t)/2 & \Delta(t) + S(t)
\end{bmatrix}
\]

Two-photon Rabi: \( \Omega(t) = \alpha_{12} \mathcal{E}^2(t) \equiv \Omega_0 \Lambda(t) \)

Shape: \( \Lambda(t) = e^{-(t/T)^2} \)

Stark shift: \( S(t) = (\alpha_{22} - \alpha_{11}) \mathcal{E}^2(t) \equiv S_0 \Lambda(t) \)

Level lines: \( \Delta(t) = \Omega_0 \sqrt{1 - \Lambda^2(t)} - S_0 \Lambda(t) \)

![Diagram showing level lines and field parameters over time]

- P_1 and P_2 populations over time
- Eigenvalues over time
- Field parameters over time
Adiabatic control of multiphoton processes (dynamical Stark shifts)

\[ H[\Omega(t),\Delta(t)] = \begin{bmatrix} 0 & \Omega(t)/2 \\ \Omega(t)/2 & \Delta(t) + S(t) \end{bmatrix} \]
Stimulated Raman adiabatic passage (STIRAP)

RWA:

\[ K_{\text{eff}}[\Omega_P(t),\Omega_S(t)] = \frac{\hbar}{2} \begin{bmatrix} 0 & \Omega_P(t) & 0 \\ \Omega_P(t) & 2\Delta_P(t) & \Omega_S(t) \\ 0 & \Omega_S(t) & 2\delta \end{bmatrix} \]

\[ \delta = \Delta_P - \Delta_S \]

Counterintuitive pulses + adiabatic passage

\[ \Rightarrow \text{complete population transfer to } |3; -1, 1\rangle \]

Properties for \( \delta = \Delta_P = 0 \):
- transfer state = dark state

No ultrahigh fidelity with Gaussian pulses
Alternative technique:
Shaping the field amplitude only
Vibration of a Morse potential (HF molecule)
Optimal state-selective adiabatic passage of an aligned molecule

Composite adiabatic passage


Rabi pulses
(no chirp)

Adiabatic pulses
(with chirp)

Universal!
Inhomogeneous broadening

But not fast!
Robust control by shaped pulses


\[
H_{\alpha, \delta}(t) = \frac{\hbar}{2} \begin{bmatrix} -\Delta(t) & \Omega(t) \\ \Omega(t) & \Delta(t) \end{bmatrix} + \frac{\hbar}{2} \begin{bmatrix} -\delta & \alpha \Omega(t) \\ \alpha \Omega(t) & \delta \end{bmatrix}
\]

\[
\phi = \begin{bmatrix} e^{i\varphi/2} \cos(\theta/2) \\ e^{-i\varphi/2} \sin(\theta/2) \end{bmatrix} e^{-i\gamma/2}
\]

\[
|\langle \phi_T|\phi_{\alpha, \delta}(t_f)\rangle|^2 = 1 + (O_1 + \bar{O}_1) + (O_2 + \bar{O}_2 + \bar{O}_1 O_1)
+ (O_3 + \bar{O}_3 + \bar{O}_1 O_2 + O_1 \bar{O}_2) + \cdots
\]

\[
O_2 + \bar{O}_2 + \bar{O}_1 O_1 = -\left|\int_{t_i}^{t_f} f(t) dt\right|^2
\]

\[
f = \langle \phi_0|V|\phi_\perp\rangle = \frac{1}{2}[\delta \sin \theta + \alpha \Omega(\cos \theta \cos \varphi - i \sin \varphi)] e^{i\gamma}
\]
\[
|\langle \phi_T | \phi_{\alpha, \delta(t_f)} \rangle|^2 = 1 + (O_1 + O_1) + (O_2 + O_2 + O_1O_1) \\
+ (O_3 + O_3 + O_1O_2 + O_1O_2) + \cdots
\]
Robust control by shaped pulses

\[ \phi = \begin{bmatrix} e^{i\varphi/2} \cos(\theta/2) \\ e^{-i\varphi/2} \sin(\theta/2) \end{bmatrix} e^{-i\gamma/2} \]

Population transfer from ground state achieved for:

\[ \theta(t) = \pi \left( \text{erf}(t) + 1 \right)/2 \]

i.e. from 0 to pi

Tracking techniques!

Parameterization for robustness:

\[ \gamma(\theta) = 2\theta + C_1 \sin(2\theta) + \cdots + C_n \sin(2n\theta) + \cdots \]

\[ \gamma(\theta) = \theta + C_1 \sin(2\theta) + \cdots + C_n \sin(2n\theta) + \cdots \]

TDSE

\[ \begin{align*}
\dot{\theta} &= \Omega \sin \varphi \\
\dot{\varphi} &= \Delta + \Omega \cos \varphi \cot \theta \\
\dot{\gamma} &= \frac{\cos \varphi}{\sin \theta} = \frac{\dot{\theta} \cot \varphi}{\sin \theta}
\end{align*} \]
Robust control by shaped pulses

<table>
<thead>
<tr>
<th>Type</th>
<th>Param.</th>
<th>Order</th>
<th>$C_1$</th>
<th>$C_2$</th>
<th>$C_3$</th>
<th>Pulse area ($\times \pi$)</th>
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<tr>
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- **Population transfer $P_2$**
  - Top graph: Population transfer $P_2$ as a function of $\alpha$.
  - Bottom graph: Logarithmic scale of $1-P_2$ vs $\alpha$.

- **$\Omega(t)$**
  - Graph showing $\Omega(t)$ vs $t$ in units of $T$.
Robust control by shaped pulses

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The graphs on the right show the behavior of the system over time, with different shapes representing different parameters and orders.
Comparison superPLAP with robust solution

SuperPLAP
Topology of dressed eigenenergies: allows the robust navigation in the dressed Hilbert space with two control parameters (shaped phase and amplitude)

Technique of (fast) parallel adiabatic passage
Application for quantum information processing (ultrafast and ultrahigh fidelity)

Composite and piecewise adiabatic passage

Robust control by shaped pulses: robust, high fidelity, fast

(PhD: M. Sala)

Vibrational and rotational control: Artificial (field-free) quantum rotor by shaping of angular wavepackets
= angular wavepacket delocalized along several axes
+ global motion of rotation
Production by adiabatic passage

Control of the shape of single-photon and multi-photon states leaking out from a cavity – atom – shaped pulse system

SU(N) arbitrary gate by adiabatic passage
Matter Driven by Light on the Ultra-Fast Lane

The FastQuast Project is a joint effort of 13 University Laboratories and Industries in the high-tech field of *Ultrafast control of quantum systems by strong laser fields* aiming mainly at training early stage researchers. FastQuast addresses the control of atoms and molecules by laser at the femtosecond scale; it covers the handling of molecules, ultrafast spectroscopy and microscopy, control of chemical reactions, production of attosecond XUV pulses, ultrafast quantum information processing, manipulation and characterization of single-photon ultrashort pulses, ultrafast quantum memory, laser filamentation, and the production of new versatile robust tunable UV ultrashort sources.

Recent highlights concern:
Quantum entanglement of Macroscopic Diamonds
Ionization and quantum cogwheel from aligned molecule
Two-colour control of high harmonic generation
Spatio-temporal characterization of UV shaped pulses
Dynamics and control in excited molecular states
High rate 3-D resolved CARS microscopy
Mirror images of light and matter
Energetic XUV super-continua
All-Kerr driven laser filamentation
Ultrafast composite adiabatic passage

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