Note: these answers have been generated by a machine algebra system called *Mathematica*. We will cover it in due course, but you may want to install and try running it now - it is really useful.

Problem 1

In[5]:= $(2 + 3\,i) + (4 - 5\,i)$

Out[5]= $6 - 2\,i$

In[6]:= $(5 + 3\,i)\,(3 - i)$

Out[6]= $18 + 4\,i$

In[7]:= $(1 - 3\,i)^2$

Out[7]= $-8 - 6\,i$

In[8]:= $(1 - 3\,i)\,(1 + 3\,i)$

Out[8]= 10

Problem 2

In[9]:=
```
z = 3 - 2 i;
```

In[10]:=
```
Conjugate[z]
```

Out[10]= $3 + 2\,i$

In[11]:=
```
z Conjugate[z]
```

Out[11]= 13

- Re[z] = $\frac{z + z^*}{2}$
- Im[z] = $\frac{z - z^*}{2i}$

Problem 3

In[12]:= $\frac{1 - i}{1 + i}$

Out[12]= $-i$

In[13]:= $\frac{1}{5 + 3\,i}$

Out[13]= $\frac{5}{34} - \frac{3\,i}{34}$

In[14]:= $\frac{3 + 2\,i}{3 - 2\,i}$

Out[14]= $\frac{5}{13} + \frac{12\,i}{13}$

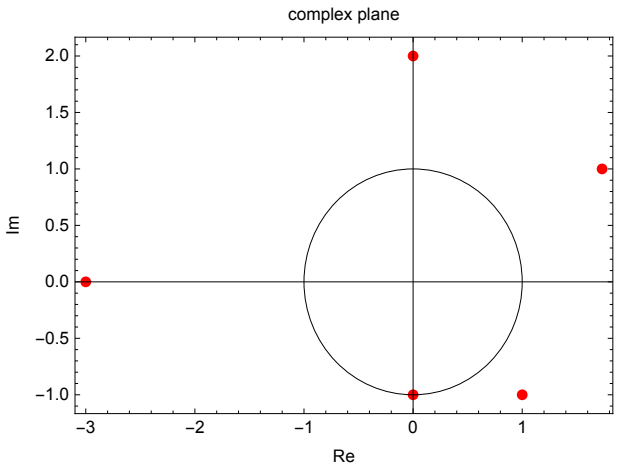
In[15]:=
$$\frac{1}{5} - \frac{3 - 4\,i}{3 + 4\,i}$$

Out[15]=
$$\frac{12}{25} + \frac{24\,i}{25}$$

Problem 4

In[16]:=
```
numbers = {2 i, -3, 1 - i, Sqrt[3] + i, 1 / i};
Show[Argand[numbers], Graphics@Circle[{0, 0}, 1]]
```

Out[17]=



In[18]:=
```
AbsArg[numbers]
```

Out[18]=
$$\left\{\left\{2, \frac{\pi}{2}\right\}, \{3, \pi\}, \left\{\sqrt{2}, -\frac{\pi}{4}\right\}, \left\{2, \frac{\pi}{6}\right\}, \left\{1, -\frac{\pi}{2}\right\}\right\}$$

In[19]:=
```
PolarForm[numbers]
```

Out[19]=
$$\left\{2\,e^{\frac{i\,\pi}{2}}, 3\,e^{i\,\pi}, \sqrt{2}\,e^{-\frac{i\,\pi}{4}}, 2\,e^{\frac{i\,\pi}{6}}, e^{-\frac{i\,\pi}{2}}\right\}$$

Problem 5

In[20]:=
```
ReIm[3 x^2 + (1 + 2 i) x + 2 (i - 1)] // Simplify
```

Out[20]=
$$\left\{-2 + x + 3\,x^2,\ 2\,(1 + x)\right\}$$

In[21]:=
```
Solve[{-2 + x + 3 x^2 == 0, 2 (1 + x) == 0}, x]
```

Out[21]=
$$\{\{x \to -1\}\}$$

In[22]:=
```
f = 3 x^2 + (1 + 2 i) x + 2 (i - 1);
f Conjugate[f] // Simplify // Expand
```

Out[23]=
$$8 + 4\,x - 7\,x^2 + 6\,x^3 + 9\,x^4$$

Problem 6

In[24]:=
```
numbers = {1 - i, Sqrt[3] + i, 2 i, -3};
```

In[25]:=
```
PolarForm[numbers]
```

Out[25]=
$$\left\{\sqrt{2}\ e^{-\frac{i\pi}{4}},\ 2\,e^{\frac{i\pi}{6}},\ 2\,e^{\frac{i\pi}{2}},\ 3\,e^{i\pi}\right\}$$

In[26]:=
```
PolarForm[Conjugate[numbers]]
```

Out[26]=
$$\left\{\sqrt{2}\ e^{\frac{i\pi}{4}},\ 2\,e^{-\frac{i\pi}{6}},\ 2\,e^{-\frac{i\pi}{2}},\ 3\,e^{i\pi}\right\}$$

In[27]:=
```
PolarForm[1 / numbers]
```

Out[27]=
$$\left\{\frac{e^{\frac{i\pi}{4}}}{\sqrt{2}},\ \frac{1}{2}e^{-\frac{i\pi}{6}},\ \frac{1}{2}e^{-\frac{i\pi}{2}},\ \frac{e^{i\pi}}{3}\right\}$$

Problem 7

In[28]:=
```
Solve[{e^(i φ) == Cos[φ] + i Sin[φ], e^(-i φ) == Cos[φ] - i Sin[φ]}, {Cos[φ], Sin[φ]}] // Expand
```

Out[28]=
$$\left\{\left\{\text{Cos}[\varphi] \to \frac{e^{-i\varphi}}{2} + \frac{e^{i\varphi}}{2},\ \text{Sin}[\varphi] \to \frac{1}{2}\,i\,e^{-i\varphi} - \frac{1}{2}\,i\,e^{i\varphi}\right\}\right\}$$

Problem 8

In[29]:=
```
ψ = C e^(i n φ);
Solve[{∫_0^(2 π) Conjugate[ψ] ψ dφ == 1, C > 0}, C]
```

Out[30]=
$$\left\{\left\{C \to \frac{1}{\sqrt{2\,\pi}}\right\}\right\}$$

In[31]:=
```
∫_0^(2 π) Conjugate[C e^(i n φ)] C e^(i m φ) dφ
```

Out[31]=
$$0$$

Problem 9

In[32]:=
```
∫_0^∞ e^(-x) Cos[2 x] dx
```

Out[32]=
$$\frac{1}{5}$$

In[33]:=
```
∫_0^∞ e^(-2 x) Sin[x]^3 dx
```

Out[33]=
$$\frac{6}{65}$$