Week 1 workshop exercises

1. Open the brackets and express as a single complex number:

$$(2+3i)+(4-5i) \qquad (5+3i)(3-i) \qquad (1-3i)^2 \qquad (1-3i)(1+3i)$$

2. If $z=3-2i$:

   (a) find z^*;

   (b) find zz^*;

   (c) express the real and the imaginary parts of z in terms of z and z^*.

3. Perform the division and express as a single complex number:

$$\frac{1-i}{1+i} \qquad \frac{1}{5+3i} \qquad \frac{3+2i}{3-2i} \qquad \frac{1}{5}-\frac{3-4i}{3+4i}$$

4. Plot as a point on the complex plane, find the amplitude and the phase, and express in polar form:

$$2i \qquad -3 \qquad 1-i \qquad \sqrt{3}+i \qquad 1/i$$

5. Given the complex function $f(x)=3x^2+(1+2i)x+2(i-1)$,

   (a) express it in the form $f(x)=g(x)+ih(x)$, where $g(x)$ and $h(x)$ are real functions;

   (b) solve $g(x)=0$ and $h(x)=0$, and hence $f(x)=0$;

   (c) find $|f(x)|^2$.

6. Express (a) z; (b) z^*; (c) z^{-1} in the exponential form $z=Ae^{i\varphi}$:

$$1-i \qquad \sqrt{3}+i \qquad 2i \qquad -3$$

7. Derive the reciprocal relations in Equation (11) of the lecture notes from Equation (10). Proceed by writing Euler's formula for $e^{i\varphi}$ and $e^{-i\varphi}$, and then adding or subtracting those expressions.

8*. Quantum mechanical wavefunctions for a rotating methyl group are $\psi_n(\varphi)=Ce^{in\varphi}$, where n is an integer and C is a real number called a *normalisation constant*.

   (a) Calculate the normalisation constant for which

$$\int_0^{2\pi} |\psi_n(\varphi)|^2 \, d\varphi = 1$$

   (b) Demonstrate that

$$\int_0^{2\pi} \psi_m^*(\varphi)\psi_n(\varphi)\, d\varphi = 0 \quad \text{if} \quad m \neq n$$

9*. Use the formulae you have derived in P7 to eliminate trigonometric functions and take the integrals:

$$\int_0^{\infty} e^{-x}\cos(2x)\,dx \qquad \int_0^{\infty} e^{-2x}\sin^3(x)\,dx$$