Week 1 workshop exercises

1. Open the brackets and express as a single complex number:
\[
(2 + 3i) + (4 - 5i) \quad (5 + 3i)(3 - i) \quad (1 - 3i)^2 \quad (1 - 3i)(1 + 3i)
\]

2. If \( z = 3 - 2i \):
   (a) find \( z^* \);
   (b) find \( zz^* \);
   (c) express the real and the imaginary parts of \( z \) in terms of \( z \) and \( z^* \).

3. Perform the division and express as a single complex number:
\[
\frac{1 - i}{1 + i} \quad \frac{1}{5 + 3i} \quad \frac{3 + 2i}{3 - 2i} \quad \frac{1}{5} \quad \frac{3 - 4i}{3 + 4i}
\]

4. Plot as a point on the complex plane, find the amplitude and the phase, and express in polar form:
\[
2i \quad -3 \quad 1 - i \quad \sqrt{3} + i \quad 1/i
\]

5. Given the complex function \( f(x) = 3x^2 + (1 + 2i)x + 2(i - 1) \),
   (a) express it in the form \( f(x) = g(x) + ih(x) \), where \( g(x) \) and \( h(x) \) are real functions;
   (b) solve \( g(x) = 0 \) and \( h(x) = 0 \), and hence \( f(x) = 0 \);
   (c) find \( |f(x)|^2 \).

6. Express (a) \( z \); (b) \( z^* \); (c) \( z^{-1} \) in the exponential form \( z = Ae^{i\phi} \):
\[
1 - i \quad \sqrt{3} + i \quad 2i \quad -3
\]

7. Derive the reciprocal relations in Equation (11) of the lecture notes from Equation (10). Proceed by writing Euler’s formula for \( e^{i\phi} \) and \( e^{-i\phi} \), and then adding or subtracting those expressions.

8*. Quantum mechanical wavefunctions for a rotating methyl group are \( \psi_n(\phi) = Ce^{i\omega \phi} \), where \( n \) is an integer and \( C \) is a real number called a normalisation constant.
   (a) Calculate the normalisation constant for which
\[
\int_{0}^{2\pi} |\psi_n(\phi)|^2 d\phi = 1
\]
   (b) Demonstrate that
\[
\int_{0}^{2\pi} \psi_m^*(\phi)\psi_n(\phi) d\phi = 0 \quad \text{if} \quad m \neq n
\]

9*. Use the formulae you have derived in P7 to eliminate trigonometric functions and take the integrals:
\[
\int_{0}^{\infty} e^{-x} \cos(2x) dx \quad \int_{0}^{\infty} e^{-2x} \sin^3(x) dx
\]