

Week 3 workshop exercises

1. Differentiate using limits: (a) $2x^2 + 3x + 4$ (b) x^4
2. Differentiate using the table of simple derivatives:
(a) x^3 (b) $x^{5/4}$ (c) $x^{1/3}$ (d) $1 - 2x + 3x^2 - 4x^3 + 5 \sin x - 6 \cos x + 7e^x - 8 \ln x$
3. An approximate equation of state for a non-ideal gas at low pressure is:

$$pV = nRT \left(1 - \frac{nB}{V} \right)$$

where B is an experimentally determined constant. Solve for P and find $\partial p / \partial V$.

4. Differentiate using product, quotient, and chain rules:

(a) $(1 - 4x^2) \cos x$ (b) $(2 + 3x)e^x$ (c) $e^x \cos x$ (d) $x \ln x$ (e) $(1 + x)^5$
(f) $\sqrt{2 + x^2}$ (g) e^{-2x} (ö) $\cos(2x^2 - 3x + 1)$ (ξ) $e^{\sin x}$ (η) $\ln[(2 + x)/(3 - x)]$
(ю) $\ln(\sin 2x + \sin^2 x)$ (ж) $\tan(4x) \cos^2(2x)$ (æ) $x^2 e^{2x^2 + 3}$

5. Find $\partial z / \partial x$ and $\partial z / \partial y$ for:

(a) $z = 2x^2 - y^2$ (b) $z = \sin(x^2 - y^2)$

6. Find all non-zero partial derivatives of:

(a) $z = x^2 - 3x^2y + 4xy^2$ (b) $u = 3x^2 + y^2 + 2xy^2$

7. Find all first and second partial derivatives of $z = 2x^2y + \cos(x + y)$.

8. Show that $f''_{xy} = f''_{yx}$ for: (a) $f = x^3 - 3x^2y + y^3$ (b) $f = x^2 \cos(y - x)$

9. Show that $f'''_{xyz} = f'''_{yzx} = f'''_{zxy}$ for $f = \cos(x + 2y + 3z)$.

10. Show that the solutions of the one-dimensional wave equation

$$\frac{\partial^2 \varphi}{\partial x^2} = \frac{1}{c^2} \frac{\partial^2 \varphi}{\partial t^2}$$

where c is the speed of light, have the following general form:

$$\varphi = f(x - ct) + g(x + ct)$$

where f and g are arbitrary well-behaved functions.