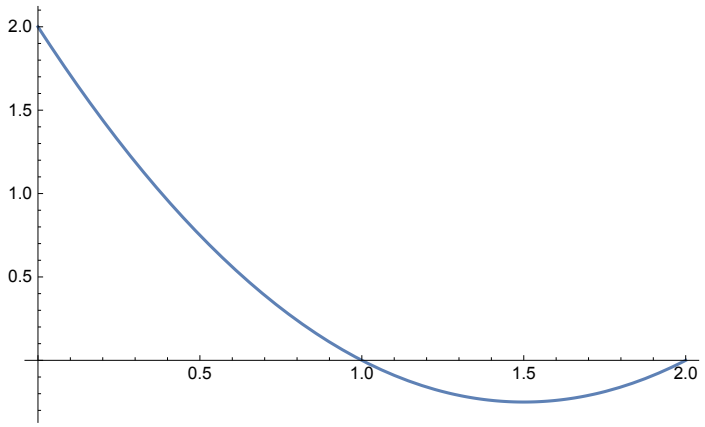
W4 - P1a

```
Solve[∂x (x^2 - 3 x + 2) == 0, x]
Plot[x^2 - 3 x + 2, {x, -0, 2}]
```

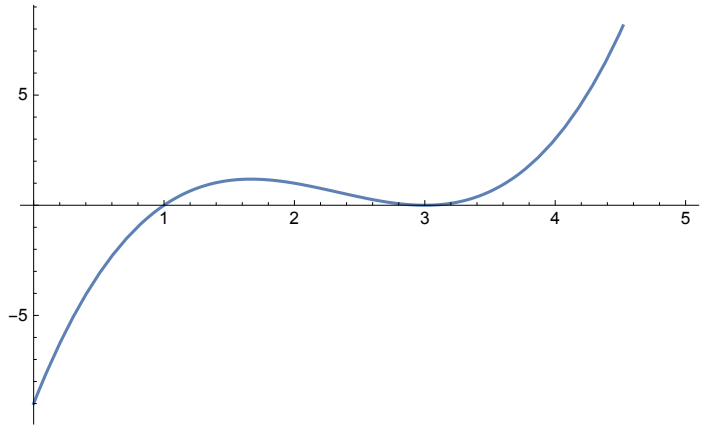
$$\left\{\left\{x \to \frac{3}{2}\right\}\right\}$$



W4 - P1b

```
Solve[∂x (x^3 - 7 x^2 + 15 x - 9) == 0, x]
Plot[x^3 - 7 x^2 + 15 x - 9, {x, -0, 5}]
```

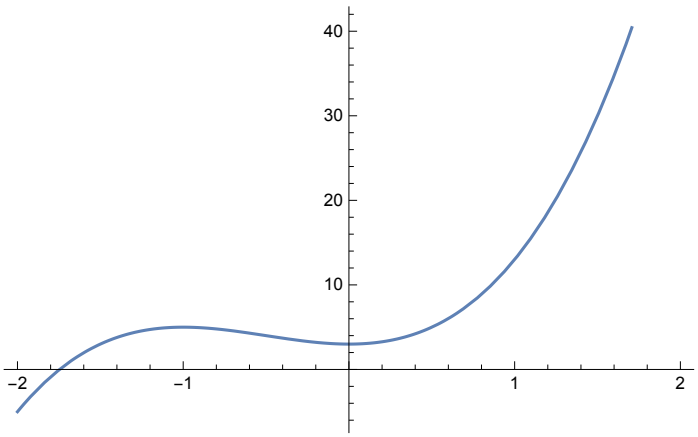
$$\left\{\left\{x \to \frac{5}{3}\right\}, \{x \to 3\}\right\}$$



W4 - P1c

```
Solve[∂x (4 x^3 + 6 x^2 + 3) == 0, x]
Plot[4 x^3 + 6 x^2 + 3, {x, -2, 2}]
```

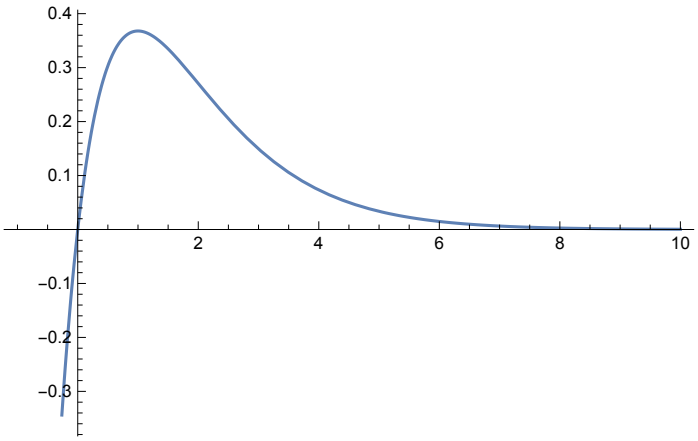
```
{{x → -1}, {x → 0}}
```



W4 - P1d

```
Solve[∂x (x Exp[-x]) == 0, x]
Plot[x Exp[-x], {x, -1, 10}]
```

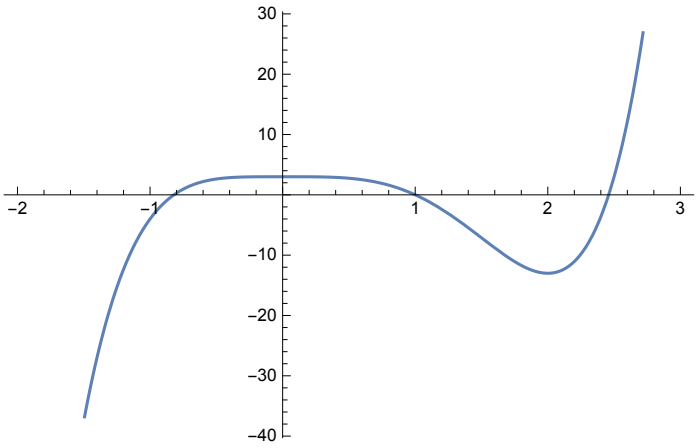
```
{{x → 1}}
```



W4 - P1e

```
Solve[∂x (2 x^5 - 5 x^4 + 3) == 0, x]
Plot[2 x^5 - 5 x^4 + 3, {x, -2, 3}]
```

```
{{x → 0}, {x → 0}, {x → 0}, {x → 2}}
```



W4 - P2

```
∂x (x^3 - 7 x^2 + 16 x - 10) /. {x → 2}
∂x,x (x^3 - 7 x^2 + 16 x - 10) /. {x → 2}
```

```
0
```

```
-2
```

```
∂x (x^3 - 7 x^2 + 16 x - 10) /. {x → 8/3}
∂x,x (x^3 - 7 x^2 + 16 x - 10) /. {x → 8/3}
```

```
0
```

```
2
```

W4 - P3

```
$Assumptions = a > 0 && b > 0;
Minimize[a/r^12 - b/r^6, r] // FullSimplify
```

$$\left\{-\frac{b^2}{4\,a},\ \left\{r \to -2^{1/6}\left(\frac{a}{b}\right)^{1/6}\right\}\right\}$$

```
Solve[{r_e == 2^(1/6) (a/b)^(1/6), U_B == b^2/(4 a)}, {a, b}]
```

$$\left\{\left\{a \to r_e^{12}\,U_B,\ b \to 2\,r_e^6\,U_B\right\}\right\}$$

```
a/r^12 - b/r^6 /. {a → U_B r_e^12, b → 2 U_B r_e^6}
```

$$-\frac{2\,r_e^6\,U_B}{r^6} + \frac{r_e^{12}\,U_B}{r^{12}}$$

W4 - P4

```
$Assumptions = k > 0 && T > 0 && m > 0;
p[v_] := 4 π (m / (2 π k T))^(3/2) v^2 Exp[- m v^2 / (2 k T)];
Solve[∂_v p[v] == 0, v] // FullSimplify
```

$$\left\{\{v \to 0\}, \left\{v \to -\sqrt{2}\sqrt{\frac{k\,T}{m}}\right\}, \left\{v \to \sqrt{2}\sqrt{\frac{k\,T}{m}}\right\}\right\}$$

```
Sqrt[2 k T / m] /. {k → 1.380 × 10^-23, T → 298, m → 0.028 / (6.022 × 10^23)}
```

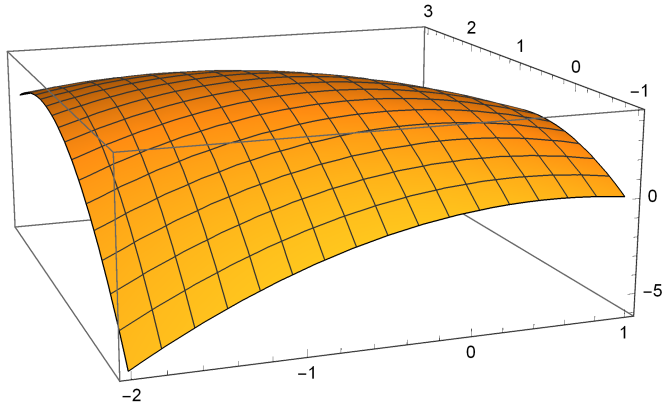
```
420.585
```

W4 - P5a

```
f[x_, y_] := 3 - x^2 - x y - y^2 + 2 y;
Solve[{∂_x f[x, y] == 0, ∂_y f[x, y] == 0}, {x, y}]
```

$$\left\{\left\{x \to -\frac{2}{3}, y \to \frac{4}{3}\right\}\right\}$$

```
Plot3D[f[x, y], {x, -2, 1}, {y, -1, 3}]
```

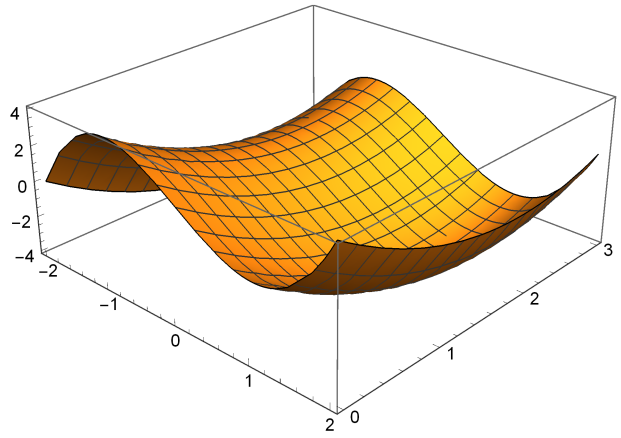


W4 - P5b

```
f[x_, y_] := x^3 + y^2 - 3 x - 4 y + 2;
Solve[{∂_x f[x, y] == 0, ∂_y f[x, y] == 0}, {x, y}]
```

$$\{\{x \to -1, y \to 2\}, \{x \to 1, y \to 2\}\}$$

```
Plot3D[f[x, y], {x, -2, 2}, {y, 0, 3}]
```



Least squares formulae

```
a[x_, y_, n_] :=
  (n Sum[(x[[k]] y[[k]]), {k, 1, n}] - (Sum[x[[k]], {k, 1, n}]) (Sum[y[[k]], {k, 1, n}]))/(n Sum[(x[[k]]^2), {k, 1, n}] - (Sum[x[[k]], {k, 1, n}])^2);
b[x_, y_, n_] := ((Sum[y[[k]], {k, 1, n}]) (Sum[(x[[k]]^2), {k, 1, n}]) - (Sum[x[[k]], {k, 1, n}]) (Sum[(x[[k]] y[[k]]), {k, 1, n}]))/
  (n Sum[(x[[k]]^2), {k, 1, n}] - (Sum[x[[k]], {k, 1, n}])^2);
```

W4 - P6a, BCA

```
x = {0.00, 0.10, 0.25, 0.50, 0.75, 1.25, 2.50};
y = {0.000, 0.046, 0.136, 0.209, 0.299, 0.484, 0.862};
a[x, y, 7]
b[x, y, 7]
```

```
0.340944
```

```
0.0302786
```

W4 - P6a, Bradford

```
x = {0.00, 0.10, 0.25, 0.50, 0.75, 1.25, 2.50};
y = {0.000, 0.039, 0.123, 0.182, 0.261, 0.409, 0.620};
a[x, y, 7]
b[x, y, 7]
```

```
0.246021
```

```
0.0453979
```

W4 - P6b, BCA

```
y = {0.00, 0.10, 0.25, 0.50, 0.75, 1.25, 2.50};
x = {0.000, 0.046, 0.136, 0.209, 0.299, 0.484, 0.862};
a[x, y, 7]
b[x, y, 7]
```

```
2.91618
```

```
-0.0839064
```

W4 - P6b, Bradford

```
y = {0.00, 0.10, 0.25, 0.50, 0.75, 1.25, 2.50};
x = {0.000, 0.039, 0.123, 0.182, 0.261, 0.409, 0.620};
a[x, y, 7]
b[x, y, 7]
```

```
3.93473
```

```
-0.154193
```

W4 - P7, using Bradford result from P6b

```
c[a_] := 3.93473 * a - 0.15419;
c[0.118]
c[0.119]
c[0.156]
c[0.155]
```

```
0.310108
```

```
0.314043
```

```
0.459628
```

```
0.455693
```

W4 - P8

```
x = {1.0, 2.0, 3.0, 4.0, 5.0, 6.0, 7.0, 8.0, 9.0, 10.0, 11.0, 12.0};
y = {4.4, 4.9, 6.4, 7.3, 8.8, 10.3, 11.7, 13.2, 14.8, 15.3, 16.5, 17.2};
a[x, y, 12]
b[x, y, 12]
```

```
1.25734
```

```
2.72727
```