## CHEM1047 – Week 5 problem set

1. Using Newton-Raphson method, find all roots of the following equations within the indicated intervals to four decimal places:

   (a) $1-x=2\sin x$, all real $x$

   (b) $x^2=2+\ln x$, $0<x\le 2$

   (c) $2\sin x=\cos 2x$, $0\le x<2\pi$

   (d) $e^{-x}=\tan x$, $0\le x<2\pi$

2. Using Newton-Raphson method, find all stationary points of the following functions within the indicated intervals to four decimal places, and determine their type:

   (a) $f(x)=e^{-x}\ln(x)$, $x>1$

   (b) $f(x)=\cos\left(x^2+3\right)$, $0<x\le 2$

3. Calculate differentials of the following functions:

   (a) $y=2x$

   (b) $y=3x^2+2x+1$

   (c) $y=\sin x$

4. The volume of a sphere of radius $r$ is $V(r)=4\pi r^3/3$. Calculate the differential $dV$ and explain its geometric meaning.

5. Find the total differentials of the following functions:

   (a) $f(x,y)=x^2+y^2$

   (b) $f(x,y)=3x^2+\sin(x-y)$

   (c) $f(x,y)=x^3y^2+\ln y$

   (d) $f(r,\theta,\varphi)=r\sin\theta\sin\varphi$

6. Test the following differentials for exactness:

   (a) $(4x+3y)\,dx+(3x+8y)\,dy$

   (b) $y\cos(x)\,dx+\sin(x)\,dy$

7. Given the total differential of the Gibbs free energy $dG=-SdT+VdP$, show that:

$$\frac{\partial S(P,T)}{\partial P}=\frac{\partial V(P,T)}{\partial T}$$