CHEM1047 - Week 10 - Workshop problem set

1. Using the suggested substitutions, evaluate the integrals:
   (a) \( \int (3x+1)^5 \, dx \), \( u = 3x + 1 \)  
   (b) \( \int (3y^2 + 2)e^{(y^3+2y)} \, dy \), \( u = y^3 + 2y \)

2. Find an appropriate substitution and evaluate the following integrals:
   (a) \( \int \sqrt{2x - 1} \, dx \)  
   (b) \( \int (1 - \alpha)e^{4\alpha - 2\alpha^2} \, d\alpha \)  
   (c) \( \int \cos(\varphi)e^{2\sin(\varphi)} \, d\varphi \)

3. The rotational partition function of an ensemble of linear molecules with rotational temperature \( \theta_R \)

\[
Q = \sum_{J=0}^{\infty} (2J + 1)e^{\frac{J(J+1)\theta_R}{\hbar}}
\]

may be approximated by replacing the sum (for which no convenient expression exists) with an integral, which is quite easy to take:

\[
Q \approx \int_0^{\infty} (2J + 1)e^{\frac{J(J+1)\theta_R}{\hbar}} \, dJ
\]

Take this integral.

4. Evaluate the following integrals using substitutions:
   (a) \( \int_1^{2} \frac{xdx}{3x^2 - 2} \)  
   (b) \( \int_0^{\pi} \frac{\sin(\sqrt{x + \pi})}{\sqrt{x}} \, dx \)  
   (c) \( \int_0^{\pi/2} \frac{\sin(\theta) \cos(\theta)}{\theta} \, d\theta \)

   (d) \( \int_0^{1} \frac{dx}{\sqrt{2-x^2}} \)  
   (e) \( \int_0^{\infty} xe^{-x^2} \, dx \)

5. Evaluate the following integrals using integration by parts:
   (a) \( \int x \sin(x) \, dx \)  
   (b) \( \int x^3 \sin(x) \, dx \)  
   (c) \( \int (x+1)^2 \cos(2x) \, dx \)  
   (d) \( \int x^2 e^{2x} \, dx \)

   (e) \( \int xe^x \, dx \)  
   (f) \( \int x^2 e^{-2x} \, dx \)  
   (g) \( \int x \ln(x) \, dx \)  
   (h) \( \int \ln\frac{x}{x^2} \, dx \)

6. Good luck trying to evaluate \( \int \frac{dx}{\cos x} \).