

CHEM1047 - Week 11 - Workshop problem set

1. Use the substitution $u = x + \sqrt{x^2 + a^2}$ to show that

$$\int \frac{dx}{\sqrt{x^2 + a^2}} = \ln \left[x + \sqrt{x^2 + a^2} \right] + C$$

2. The probability density $p(v)$ of molecular velocities in a gas with molecules of mass m at temperature T is given by the Maxwell-Boltzmann distribution:

$$p(v) = 4\pi \left(\frac{m}{2\pi kT} \right)^{3/2} v^2 \exp \left(-\frac{mv^2}{2kT} \right)$$

where k is Boltzmann's constant. Find the average velocity:

$$\langle v \rangle = \int_0^{\infty} vp(v) dv$$

3. By differentiating the integral

$$\int_0^{\infty} e^{-ax^2} dx = \frac{1}{2} \sqrt{\frac{\pi}{a}}$$

repeatedly with respect to the parameter a , demonstrate that:

$$\int_0^{\infty} x^{2n} e^{-ax^2} dx = \frac{1 \cdot 3 \cdot 5 \cdots (2n-1)}{2^{n+1} a^n} \sqrt{\frac{\pi}{a}}$$

4. Using the substitution $t = \tan(\theta/2)$, evaluate the following integrals:

(a) $\int \frac{d\theta}{\cos(\theta)}$

(b) $\int \frac{d\theta}{5 - 3\cos(\theta)}$

(c) $\int \frac{d\theta}{1 + \sin\theta + \cos\theta}$

5. Using the Taylor series method, evaluate the accuracy of the following finite difference approximation for the second derivative of the function $f(x)$:

$$f''(x) \approx \frac{f(x+h) - 2f(x) + f(x-h)}{h^2}$$

Proceed by building Taylor series for $f(x \pm h)$ with respect to h and combining them to isolate the second derivative term.

6. Way back, in the era when computer games were still intellectually sophisticated, the following problem was offered by a genie in Baldur's Gate 2. *A princess is as old as the prince will be when the princess is twice as old as the prince was when the princess' age was half the sum of their current ages. Which of the following could be true?*

(a) princess is 30, prince is 20

(b) princess is 30, prince is 40

(c) princess is 40, prince is 30

(d) princess is 20, prince is 30

(e) they are both the same age