

Problem I

In[1]:= **Unprotect[C];**

$$A = \begin{pmatrix} 1 & -2 & 3 \\ 0 & 3 & 4 \end{pmatrix}; \quad B = \begin{pmatrix} 0 & 1 & -4 \\ 2 & -3 & 0 \end{pmatrix}; \quad C = \begin{pmatrix} -5 & 3 \\ 4 & -1 \\ 2 & -1 \end{pmatrix}; \quad M = \begin{pmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & -1 \end{pmatrix};$$

$$P = \begin{pmatrix} 1 & -2 \\ 0 & 4 \end{pmatrix}; \quad Q = \begin{pmatrix} 3 & 0 \\ 0 & 1 \end{pmatrix}; \quad a = \begin{pmatrix} 0 \\ -3 \\ 1 \end{pmatrix}; \quad b = (2 \ 5 \ -2);$$

In[3]:= **Det[A]**

 **Det:** Argument {{1, -2, 3}, {0, 3, 4}} at position 1 is not a non-empty square matrix.

Out[3]= **Det** [{{1, -2, 3}, {0, 3, 4}}]

In[4]:= **Tr[A]**

Out[4]= 4

In[5]:= **Det[M]**

Out[5]= -6

In[6]:= **Tr[M]**

Out[6]= 4

In[7]:= **Det[P]**

Out[7]= 4

In[8]:= **Tr[P]**

Out[8]= 5

In[9]:= **Transpose[A] // MatrixForm**

Out[9]/MatrixForm=

$$\begin{pmatrix} 1 & 0 \\ -2 & 3 \\ 3 & 4 \end{pmatrix}$$

In[10]:= **Transpose[C] // MatrixForm**

Out[10]/MatrixForm=

$$\begin{pmatrix} -5 & 4 & 2 \\ 3 & -1 & -1 \end{pmatrix}$$

In[11]:= **Transpose[M] // MatrixForm**

Out[11]/MatrixForm=

$$\begin{pmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

In[12]:= **Transpose[a] // MatrixForm**

Out[12]/MatrixForm=

$$(0 \ -3 \ 1)$$

In[13]:= **Transpose[b] // MatrixForm**

Out[13]/MatrixForm=

$$\begin{pmatrix} 2 \\ 5 \\ -2 \end{pmatrix}$$

In[14]:= **a + Transpose[b] // MatrixForm**

Out[14]//MatrixForm=

$$\begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix}$$

In[15]:= **Transpose[a] + b // MatrixForm**

Out[15]//MatrixForm=

$$\begin{pmatrix} 2 & 2 & -1 \end{pmatrix}$$

In[16]:= **A.B // MatrixForm**

 **Dot:** Tensors {{1, -2, 3}, {0, 3, 4}} and {{0, 1, -4}, {2, -3, 0}} have incompatible shapes.

Out[16]//MatrixForm=

$\{\{1, -2, 3\}, \{0, 3, 4\}\} \cdot \{\{0, 1, -4\}, \{2, -3, 0\}\}$

In[17]:= **B.C // MatrixForm**

Out[17]//MatrixForm=

$$\begin{pmatrix} -4 & 3 \\ -22 & 9 \end{pmatrix}$$

In[18]:= **C.B // MatrixForm**

Out[18]//MatrixForm=


$$\begin{pmatrix} 6 & -14 & 20 \\ -2 & 7 & -16 \\ -2 & 5 & -8 \end{pmatrix}$$

In[19]:= **C.P // MatrixForm**

Out[19]//MatrixForm=

$$\begin{pmatrix} -5 & 22 \\ 4 & -12 \\ 2 & -8 \end{pmatrix}$$

In[20]:= **P.C // MatrixForm**

 **Dot:** Tensors {{1, -2}, {0, 4}} and {{-5, 3}, {4, -1}, {2, -1}} have incompatible shapes.

Out[20]//MatrixForm=

$\{\{1, -2\}, \{0, 4\}\} \cdot \{\{-5, 3\}, \{4, -1\}, \{2, -1\}\}$

In[21]:= **M.M // MatrixForm**

Out[21]//MatrixForm=

$$\begin{pmatrix} 9 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

In[22]:= **P.Q // MatrixForm**

Out[22]//MatrixForm=

$$\begin{pmatrix} 3 & -2 \\ 0 & 4 \end{pmatrix}$$

In[23]:= **B.a // MatrixForm**

Out[23]//MatrixForm=

$$\begin{pmatrix} -7 \\ 9 \end{pmatrix}$$

In[24]:= **a.b // MatrixForm**

Out[24]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 \\ -6 & -15 & 6 \\ 2 & 5 & -2 \end{pmatrix}$$

In[25]:= **b.a // MatrixForm**

Out[25]//MatrixForm=
 (-17)

In[26]:= **Transpose[a].Transpose[b] // MatrixForm**

Out[26]//MatrixForm=
 (-17)

In[27]:= **C.a // MatrixForm**

 **Dot:** Tensors $\{-5, 3\}$, $\{4, -1\}$, $\{2, -1\}$ and $\{0, -3, 1\}$ have incompatible shapes.

Out[27]//MatrixForm=
 $\{-5, 3\}, \{4, -1\}, \{2, -1\} \cdot \{\{0\}, \{-3\}, \{1\}\}$

In[28]:= **Transpose[a].C // MatrixForm**

Out[28]//MatrixForm=
 $(-10 \ 2)$

Problem 2

In[29]:= $\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} // \text{MatrixForm}$

Out[29]//MatrixForm=
 $\begin{pmatrix} y \\ x \\ z \end{pmatrix}$

Problem 3

In[30]:= $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} // \text{MatrixForm}$

Out[30]//MatrixForm=
 $\begin{pmatrix} x \\ y \\ 0 \end{pmatrix}$

Problem 4

In[31]:= **Eigenvalues** $\left[\begin{pmatrix} 2 & 2 \\ 1 & 3 \end{pmatrix}\right]$
Eigenvectors $\left[\begin{pmatrix} 2 & 2 \\ 1 & 3 \end{pmatrix}\right] // \text{Map}[\text{Normalize}, \#] \&$

Out[31]= $\{4, 1\}$

Out[32]= $\left\{\left\{\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right\}, \left\{-\frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}}\right\}\right\}$

```
In[33]= Eigenvalues[ $\begin{pmatrix} 3 & 1 \\ 1 & 3 \end{pmatrix}$ ]
      Eigenvectors[ $\begin{pmatrix} 3 & 1 \\ 1 & 3 \end{pmatrix}$ ] // Map[Normalize, #] &
```

```
Out[33]= {4, 2}
```

```
Out[34]=  $\left\{ \left\{ \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right\}, \left\{ -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right\} \right\}$ 
```

```
In[35]= Eigenvalues[ $\begin{pmatrix} 1 & 2 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ ]
      Eigenvectors[ $\begin{pmatrix} 1 & 2 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ ] // Map[Normalize, #] &
```

```
Out[35]= {3, -1, 1}
```

```
Out[36]=  $\left\{ \left\{ \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0 \right\}, \left\{ -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0 \right\}, \{0, 0, 1\} \right\}$ 
```

```
In[37]= Eigenvalues[ $\begin{pmatrix} 3 & 1 \\ -1 & 3 \end{pmatrix}$ ]
      Eigenvectors[ $\begin{pmatrix} 3 & 1 \\ -1 & 3 \end{pmatrix}$ ] // Map[Normalize, #] &
```

```
Out[37]= {3 + i, 3 - i}
```

```
Out[38]=  $\left\{ \left\{ -\frac{i}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right\}, \left\{ \frac{i}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right\} \right\}$ 
```

Problem 5

```
In[39]= A = Eigenvectors[ $\begin{pmatrix} 3 & 1 \\ 1 & 3 \end{pmatrix}$ ];
      {A[[1]]}.Transpose[{A[[2]]}]
```

```
Out[40]= {{0}}
```

```
In[41]= A = Eigenvectors[ $\begin{pmatrix} 1 & 2 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ ];
      {A[[1]]}.Transpose[{A[[2]]}]
      {A[[2]]}.Transpose[{A[[3]]}]
      {A[[1]]}.Transpose[{A[[3]]}]
```

```
Out[42]= {{0}}
```

```
Out[43]= {{0}}
```

```
Out[44]= {{0}}
```

Problem 6

```
In[45]= ConjugateTranspose[ $\begin{pmatrix} 1+i & 2-i \\ 3+i & -i \end{pmatrix}$ ] // MatrixForm
```

```
Out[45]//MatrixForm=
 $\begin{pmatrix} 1-i & 3-i \\ 2+i & i \end{pmatrix}$ 
```

In[46]:= **HermitianMatrixQ**[$\begin{pmatrix} 1 + \mathbf{i} & 2 - \mathbf{i} \\ 3 + \mathbf{i} & -\mathbf{i} \end{pmatrix}$]

Out[46]= **False**

In[47]:= **ConjugateTranspose**[$\begin{pmatrix} 2 & \mathbf{i} \\ -\mathbf{i} & 2 \end{pmatrix}$] // **MatrixForm**

Out[47]//MatrixForm=

$$\begin{pmatrix} 2 & \mathbf{i} \\ -\mathbf{i} & 2 \end{pmatrix}$$

In[48]:= **HermitianMatrixQ**[$\begin{pmatrix} 2 & \mathbf{i} \\ -\mathbf{i} & 2 \end{pmatrix}$]

Out[48]= **True**

In[49]:= **ConjugateTranspose**[$\begin{pmatrix} \mathbf{0} & -\mathbf{i} & \mathbf{0} \\ \mathbf{i} & \mathbf{0} & \mathbf{i} \\ \mathbf{0} & -\mathbf{i} & \mathbf{0} \end{pmatrix}$] // **MatrixForm**

Out[49]//MatrixForm=

$$\begin{pmatrix} \mathbf{0} & -\mathbf{i} & \mathbf{0} \\ \mathbf{i} & \mathbf{0} & \mathbf{i} \\ \mathbf{0} & -\mathbf{i} & \mathbf{0} \end{pmatrix}$$

In[50]:= **HermitianMatrixQ**[$\begin{pmatrix} \mathbf{0} & -\mathbf{i} & \mathbf{0} \\ \mathbf{i} & \mathbf{0} & \mathbf{i} \\ \mathbf{0} & -\mathbf{i} & \mathbf{0} \end{pmatrix}$]

Out[50]= **True**