Week 19 workshop exercises

1. For the following matrices and vectors

$$\mathbf{A}=\begin{bmatrix}1 & -2 & 3\\ 0 & 3 & 4\end{bmatrix};\quad \mathbf{B}=\begin{bmatrix}0 & 1 & -4\\ 2 & -3 & 0\end{bmatrix};\quad \mathbf{C}=\begin{bmatrix}-5 & 3\\ 4 & -1\\ 2 & -1\end{bmatrix};\quad \mathbf{M}=\begin{bmatrix}3 & 0 & 0\\ 0 & 2 & 0\\ 0 & 0 & -1\end{bmatrix}$$

$$\mathbf{P}=\begin{bmatrix}1 & -2\\ 0 & 4\end{bmatrix};\quad \mathbf{Q}=\begin{bmatrix}3 & 0\\ 0 & 1\end{bmatrix};\quad \mathbf{a}=\begin{bmatrix}0\\ -3\\ 1\end{bmatrix};\quad \mathbf{b}=\begin{bmatrix}2 & 5 & -2\end{bmatrix},$$

find, if possible:

(a) $\det(\mathbf{A})$; (b) $\mathrm{tr}(\mathbf{A})$; (c) $\det(\mathbf{M})$; (d) $\mathrm{tr}(\mathbf{M})$; (e) $\det(\mathbf{P})$;

(f) $\mathrm{tr}(\mathbf{P})$; (g) $\mathbf{A}^{\mathrm{T}}$; (h) $\mathbf{C}^{\mathrm{T}}$; (i) $\mathbf{M}^{\mathrm{T}}$; (j) $\mathbf{a}^{\mathrm{T}}$; (k) $\mathbf{b}^{\mathrm{T}}$;

(l) $\mathbf{a}+\mathbf{b}^{\mathrm{T}}$; (m) $\mathbf{a}^{\mathrm{T}}+\mathbf{b}$; (n) $\mathbf{AB}$; (o) $\mathbf{BC}$; (p) $\mathbf{CB}$; (q) $\mathbf{CP}$;

(r) $\mathbf{PC}$; (s) $\mathbf{M}^2$; (t) $\mathbf{PQ}$; (u) $\mathbf{Ba}$; (v) $\mathbf{ab}$; (w) $\mathbf{ba}$;

(x) $\mathbf{a}^{\mathrm{T}}\mathbf{b}^{\mathrm{T}}$; (y) $\mathbf{Ca}$; (z) $\mathbf{a}^{\mathrm{T}}\mathbf{C}$.

2. Construct a 3×3 matrix that exchanges X and Y coordinates of the vector that it multiplies.

3. Construct a 3×3 matrix that projects a vector onto the XY plane.

4. Find eigenvalues and eigenvectors of the following matrices and normalise them:

(a) $\begin{bmatrix}2 & 2\\ 1 & 3\end{bmatrix}$; (b) $\begin{bmatrix}3 & 1\\ 1 & 3\end{bmatrix}$; (c) $\begin{bmatrix}1 & 2 & 0\\ 2 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$; (d) $\begin{bmatrix}3 & 1\\ -1 & 3\end{bmatrix}$.

5. Demonstrate that the eigenvectors of matrices 4b and 4c are orthogonal.

6. Perform the conjugate-transpose operation on the following matrices:

(a) $\begin{bmatrix}1+i & 2-i\\ 3+i & -i\end{bmatrix}$; (b) $\begin{bmatrix}2 & i\\ -i & 2\end{bmatrix}$; (c) $\begin{bmatrix}0 & -i & 0\\ i & 0 & i\\ 0 & -i & 0\end{bmatrix}$.

Which of these matrices are Hermitian?