

Week 21 workshop exercises

1. Show that the following functions are particular solutions for the specified ODEs:

(a) e^{-2x} and $e^{2x/3}$ for $3y'' + 4y' - 4y = 0$;

(b) $\cos(2x)$ and $\sin(2x)$ for $y'' + 4y = 0$.

2. Use the particular solutions to write down the general solutions for the equations in Problem 1.

3. Find the general solutions:

$$(a) \frac{d^2y}{dt^2} - \frac{dy}{dt} - 6y = 0; \quad (b) 2\frac{d^2y}{dx^2} - 8\frac{dy}{dx} + 3y = 0.$$

4. Solve the initial value problems:

$$(a) \frac{d^2x}{dt^2} + \frac{dx}{dt} - 2x = 0, \quad x(0) = 1, \quad x'(0) = 0;$$

$$(b) \frac{d^2x}{dt^2} + 6\frac{dx}{dt} + 9x = 0, \quad x(1) = 0, \quad x'(1) = 1.$$

5. Solve the boundary value problems:

$$(a) \frac{d^2y}{dx^2} + 8\frac{dy}{dx} + 16y = 0, \quad y(0) = 0, \quad y(1) = 1;$$

$$(b) \frac{d^2y}{dx^2} + 9y = 0, \quad y(0) = 0, \quad y\left(\frac{\pi}{2}\right) = 1.$$

6. Find the general solution:

$$\begin{cases} \frac{dx}{dt} = -y \\ \frac{dy}{dt} = -x \end{cases}$$