**Week 21 workshop exercises**

1. Show that the following functions are particular solutions for the specified ODEs:
   
   (a) \( e^{-2x} \) and \( e^{2x/3} \) for \( 3y'' + 4y' - 4y = 0 \);
   
   (b) \( \cos(2x) \) and \( \sin(2x) \) for \( y'' + 4y = 0 \).

2. Use the particular solutions to write down the general solutions for the equations in Problem 1.

3. Find the general solutions:
   
   (a) \( \frac{d^2 y}{dt^2} - \frac{dy}{dt} - 6y = 0 \);  
   
   (b) \( 2\frac{d^2 y}{dx^2} - 8\frac{dy}{dx} + 3y = 0 \).

4. Solve the initial value problems:

   (a) \( \frac{d^2 x}{dt^2} + \frac{dx}{dt} - 2x = 0 \), \( x(0) = 1 \), \( x'(0) = 0 \);

   (b) \( \frac{d^2 x}{dt^2} + 6\frac{dx}{dt} + 9x = 0 \), \( x(1) = 0 \), \( x'(1) = 1 \).

5. Solve the boundary value problems:

   (a) \( \frac{d^2 y}{dx^2} + 8\frac{dy}{dx} + 16y = 0 \), \( y(0) = 0 \), \( y(1) = 1 \);

   (b) \( \frac{d^2 y}{dx^2} + 9y = 0 \), \( y(0) = 0 \), \( y\left(\frac{x}{2}\right) = 1 \).

6. Find the general solution:

\[
\begin{cases}
  \frac{dx}{dt} = -y \\
  \frac{dy}{dt} = +x
\end{cases}
\]