Week 22 class problems

1. Evaluate the integral:
\[ \int_L \left[ (xy) \, dx + (2y) \, dy \right] \]
on the line \( y = 2x \) from \( x = 0 \) to \( x = 2 \).

2. Evaluate the integral:
\[ \int_L \left[ (x^2 + 2y) \, dx + (y^2 + x) \, dy \right] \]
on a curve parameterised by \( x = t \) and \( y = t^2 \) where \( t \) goes from 0 to 1.

3. Evaluate the integral:
\[ \int_0^2 \int_0^2 (x^2 + xy^2) \, dx \, dy \]
and show that the result does not depend on the order of integration.

4. Evaluate the integrals and sketch the region of integration:
\[ \int_0^{2x} \int_0^x (x^2 + y^2) \, dy \, dx \quad \int_0^{\sqrt[3]{x^2}} \int_0^x (xy^2) \, dy \, dx \]

5. Show, using a sketch of the integration region, that:
\[ \int_0^{(2-y)/2} \int_0^{2x} x^3 \, dx \, dy = \int_0^{2-2x} \int_0^y x^3 \, dx \, dy + \int_0^{(x+4)/2} \int_{-4}^0 x^3 \, dy \, dx \]
and evaluate the integral.

6. Find the total mass in the indicated volumes by integrating material density distributions:

(a) \( \rho = x^2 + y^2 + z^2 \) \quad V: the cube \( 0 \leq x \leq 1, \ 0 \leq y \leq 1, \ 0 \leq z \leq 1 \)

(b) \( \rho = xy^2z^3 \) \quad V: the box \( 0 \leq x \leq a, \ 0 \leq y \leq b, \ 0 \leq z \leq c \)

(c) \( \rho = x^2 \) \quad V: the region \( 1 - y \leq x \leq 1, \ 0 \leq y \leq 1, \ 0 \leq z \leq 2 \)

(d) \( \rho = \exp(-x - y - z) \) \quad V: the region \( 0 \leq x < \infty, \ 0 \leq y < \infty, \ 0 \leq z < \infty \)