Problem 1

In[2]:=
```
FNorm[f_] := Sqrt[∫_-1^1 Conjugate[f] * f dx];
FScal[f_, g_] := ∫_-1^1 Conjugate[f] * g dx;
```

In[4]:=
```
FNorm[x]
```

Out[4]=
$$\sqrt{\frac{2}{3}}$$

In[5]:=
```
FNorm[2^x]
```

Out[5]=
$$\sqrt{\frac{15}{\text{Log}[256]}}$$

In[6]:=
```
FNorm[Cos[π x / 2]]
```

Out[6]=
```
1
```

In[7]:=
```
FScal[1, x]
```

Out[7]=
```
0
```

In[8]:=
```
FScal[x, 2 x^2 - 1]
```

Out[8]=
```
0
```

In[9]:=
```
FScal[2 x^2 - 1, 4 x^3 - 3 x]
```

Out[9]=
```
0
```

Problem 2

In[10]:=
```
FNorm[f_] := Sqrt[∫_-π^π Conjugate[f] * f dx];
```

In[11]:=
```
A = Assuming[n ∈ Reals, Refine[FNorm[n]]]; Solve[{A == 1, n > 0}, n]
```

Out[11]=
$$\left\{\left\{n \to \frac{1}{\sqrt{2\,\pi}}\right\}\right\}$$

In[12]:=
```
A = Assuming[n ∈ Reals, Refine[FNorm[n Sin[x]]]]; Solve[{A == 1, n > 0}, n]
```

Out[12]=
$$\left\{\left\{n \to \frac{1}{\sqrt{\pi}}\right\}\right\}$$

In[13]:=
```
A = Assuming[n ∈ Reals, Refine[FNorm[Cos[2 x] / n]]]; Solve[{A == 1, n > 0}, n]
```

Out[13]=
$$\left\{\left\{n \to \sqrt{\pi}\right\}\right\}$$

In[14]:=
```
A = Assuming[n ∈ Reals, Refine[FNorm[n Sin[x]^2]]]; Solve[{A == 1, n > 0}, n]
```

Out[14]=
$$\left\{\left\{n \to \frac{2}{\sqrt{3\,\pi}}\right\}\right\}$$

In[15]:=
```
A = Assuming[n ∈ Reals, Refine[FNorm[n (i - x / π)]]]; Solve[{A == 1, n > 0}, n]
```

Out[15]=
$$\left\{\left\{n \to \frac{1}{2}\sqrt{\frac{3}{2\pi}}\right\}\right\}$$

In[16]:=
```
A = Assuming[n ∈ Reals, Refine[FNorm[n Exp[-4 i x]]]];
Solve[{A == 1, n > 0}, n]
```

Out[16]=
$$\left\{\left\{n \to \frac{1}{\sqrt{2\pi}}\right\}\right\}$$

Problem 3

In[17]:=
$$\frac{1}{\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left(\text{Exp}\left[-x^2-y^2\right]\right)\,dx\,dy$$

Out[17]= 1

In[18]:=
$$\frac{2}{\pi}\int_{-\infty}^{\infty}\left(\frac{1}{x^2+1}\right)^2 dx$$

Out[18]= 1

Problem 4

Wronskian below is a mathematical criterion for linear dependence.

In[19]:=
```
(Wronskian[{x, 1 + x, 1 - x}, x] /. {x → 1}) ≠ 0
```

Out[19]= False

In[20]:=
```
(Wronskian[{Cos[x], Cos[2 x], Cos[3 x]}, x] /. {x → 1}) ≠ 0
```

Out[20]= True

In[21]:=
```
(Wronskian[{0, x Log[x], ArcTan[x], Exp[x]}, x] /. {x → 1}) ≠ 0
```

Out[21]= False

In[22]:=
```
(Wronskian[{Log[x], Log[x^2], Log[x^3]}, x] /. {x → 1}) ≠ 0
```

Out[22]= False

Problem 5

The expansion coefficients for the three functions are in the curly brackets.

In[23]:=
```
FScal[f_, g_] := ∫_-π^π Conjugate[f] * g dx;
Thread[FScal[{Exp[-i x] / √(2 π), 1 / √(2 π), Exp[i x] / √(2 π)}, Cos[x]]]
```

Out[24]=
$$\left\{\sqrt{\frac{\pi}{2}}, 0, \sqrt{\frac{\pi}{2}}\right\}$$

In[25]:=
```
Thread[FScal[{Exp[-i x] / Sqrt[2 π], 1 / Sqrt[2 π], Exp[i x] / Sqrt[2 π]}, Sin[x]]]
```

Out[25]=
$$\left\{i \sqrt{\frac{\pi}{2}}, 0, -i \sqrt{\frac{\pi}{2}}\right\}$$

In[26]:=
```
Thread[FScal[{Exp[-i x] / Sqrt[2 π], 1 / Sqrt[2 π], Exp[i x] / Sqrt[2 π]}, 1]]
```

Out[26]=
$$\left\{0, \sqrt{2\pi}, 0\right\}$$

Problem 6

In[28]:=
```
Assuming[n ∈ Integers && k ∈ Integers && n ≠ k, 1/(2 π) ∫_(-π)^π Exp[-i n x] ∂_(x,x) Exp[i k x] 𝕕x]
```

Out[28]=
$$0$$

In[29]:=
```
Assuming[n ∈ Integers && k ∈ Integers && n == k, 1/(2 π) ∫_(-π)^π Exp[-i n x] ∂_(x,x) Exp[i k x] 𝕕x]
```

Out[29]=
$$-n^2$$