

Module VII, Lecture 01: Two-electron Dipolar Spectroscopy

There is no algebraic difference between NMR and EPR spectroscopy – the mathematical formalism that we have developed for nuclear magnetic resonance applies in exactly the same way. The table below gives NMR and EPR names for the different interaction classes, and their typical amplitudes.

Type	NMR	EPR
linear	chemical shift (kHz)	g -tensor (GHz to THz)
bilinear	dipolar, J -coupling (Hz to kHz)	dipolar, exchange, HFC (MHz)
quadratic	nuclear quadrupolar interaction (kHz to MHz)	zero-field splitting (GHz to THz)

Electron magnetogyric ratio is about 660 times greater than that of a proton, and has the opposite sign. There are cosmetic differences in the way the Zeeman interaction is written in NMR and EPR:

$$\hat{H}_Z = \mu_B \vec{B} \cdot \mathbf{g} \cdot \hat{S} \quad \hat{H}_Z = -\gamma \vec{B} \cdot (1 + \delta) \cdot \hat{S} \quad (1)$$

where μ_B is the Bohr magneton, γ is the nuclear magnetogyric ratio, \mathbf{g} is the g -tensor, and δ is the chemical shift tensor. The positive sign reflects the fact that the magnetogyric ratio of the electron is negative. Also, because electron spin interactions can be a significant perturbation to the electronic structure theory, effective interactions that are cubic and higher order in the electron spin operators can make an appearance in the zero-field splitting:

$$\hat{H}_{ZFS} = \sum_{k,q} B_{kq} \hat{O}_k^q \quad (2)$$

where \hat{O}_k^q are Stevens operators (linear combinations of irreducible spherical tensors).

Dipolar EPR spectroscopy – DEER and ESEEM

One of the primary current applications of EPR in structural biology is nanometre-scale distance measurement, which is based on the measurement of the inter-electron dipolar interaction. In classical physics, the energy of the dipolar interaction between two point dipoles is:

$$E = -\frac{\mu_0}{4\pi} \frac{3(\vec{\mu}_1 \cdot \vec{r})(\vec{r} \cdot \vec{\mu}_2) - r^2(\vec{\mu}_1 \cdot \vec{\mu}_2)}{r^5} \quad (3)$$

A significant complication in the case of EPR (which NMR spectroscopists always ignore) is that the electron magnetic moment is proportional to the g -tensor:

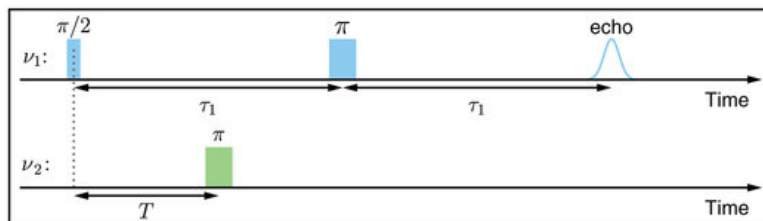
$$\hat{m} = -\frac{\partial \hat{H}}{\partial \vec{B}} = -\mathbf{g} \cdot \hat{S} \quad (4)$$

with the result that the Hamiltonian contains the usual dipolar interaction wrapped in g -tensors:

$$\hat{H}_{DD} = -\frac{\mu_0}{4\pi} \frac{\mu_B^2}{\hbar} \iint \mathbf{g}_1 \left(\frac{3(\hat{S}_1 \cdot \vec{r}_{12})(\vec{r}_{12} \cdot \hat{S}_2)}{r_{12}^5} - \frac{(\hat{S}_1 \cdot \hat{S}_2)}{r_{12}^3} \right) \mathbf{g}_2^T \rho(\vec{r}_1) \rho(\vec{r}_2) d^3\vec{r}_1 d^3\vec{r}_2 \quad (5)$$

For rigid molecules, the rotation matrices cancel and the composite $\mathbf{g}_1 \mathbf{D} \mathbf{g}_2^T$ interaction matrix still behaves as a second spherical rank interaction.

The most popular sequence that uses inter-electron dipolar interaction for distance determination is DEER. The basic DEER sequence (see *e.g.* Jeschke and Polyhach, PCCP 2007 (9) 1895-1910):



The first pulse generates the transverse magnetisation, and the objective of the other pulses is to generate a spin echo. At its maximum, all Zeeman interactions are refocused. The magnitude of the echo therefore only depends on the inter-electron interactions. An earlier version of this pulse sequence, intended for instruments that cannot pulse at two different frequencies, is called ESEEM.

Electrons interact via dipolar and exchange mechanisms. Assuming isotropic g -tensors, a two-electron system with a dipole-dipole and an exchange coupling has the following Hamiltonian:

$$\hat{H}(\Omega) = \omega_1 \hat{L}_Z^{(1)} + \omega_2 \hat{L}_Z^{(2)} - \sqrt{6} \frac{\mu_0 \gamma_1 \gamma_2 \hbar}{4\pi r^3} \sum_{m=-2}^2 \hat{T}_{2,m} D_{m,0}^{(2)}(\Omega) + J \left[\hat{L}_X^{(1)} \hat{L}_X^{(2)} + \hat{L}_Y^{(1)} \hat{L}_Y^{(2)} + \hat{L}_Z^{(1)} \hat{L}_Z^{(2)} \right] \quad (6)$$

Where Ω is a set of three Euler angles, $D_{m,0}^{(2)}(\Omega)$ are second-rank Wigner D-functions, J is defined in the NMR convention in angular frequency units and the irreducible spherical tensor operators are

$$\begin{aligned} \hat{T}_{2,2} &= +\frac{1}{2} \hat{L}_+^{(1)} \hat{L}_+^{(2)} \\ \hat{T}_{2,1} &= -\frac{1}{2} \left(\hat{L}_Z^{(1)} \hat{L}_+^{(2)} + \hat{L}_+^{(1)} \hat{L}_Z^{(2)} \right) \\ \hat{T}_{2,0} &= +\sqrt{\frac{2}{3}} \left(\hat{L}_Z^{(1)} \hat{L}_Z^{(2)} - \frac{1}{4} \left(\hat{L}_+^{(1)} \hat{L}_-^{(2)} + \hat{L}_-^{(1)} \hat{L}_+^{(2)} \right) \right) \\ \hat{T}_{2,-1} &= +\frac{1}{2} \left(\hat{L}_Z^{(1)} \hat{L}_-^{(2)} + \hat{L}_-^{(1)} \hat{L}_Z^{(2)} \right) \\ \hat{T}_{2,-2} &= +\frac{1}{2} \hat{L}_-^{(1)} \hat{L}_-^{(2)} \end{aligned} \quad (7)$$

Applying the rotating frame with respect to the Zeeman Hamiltonian

$$\hat{H}_0 = \omega_1 \hat{L}_Z^{(1)} + \omega_2 \hat{L}_Z^{(2)} \quad (8)$$

and relying on the fact that the spin echo would refocus the g -factor offset variations, we obtain the following rotating frame Hamiltonian:

$$\begin{aligned} \hat{H} &= \frac{\mu_0 \gamma_1 \gamma_2 \hbar}{4\pi r^3} \left[1 - 3 \cos^2(\theta) \right] \hat{L}_Z^{(1)} \hat{L}_Z^{(2)} + J \hat{L}_Z^{(1)} \hat{L}_Z^{(2)} = \\ &= D \left[1 - 3 \cos^2(\theta) \right] \hat{L}_Z^{(1)} \hat{L}_Z^{(2)} + J \hat{L}_Z^{(1)} \hat{L}_Z^{(2)} \end{aligned} \quad (9)$$

which, when acting on the transverse magnetization, would produce the following oscillation:

$$s(\theta, t) = \cos \left[\left(D \left[1 - 3 \cos^2(\theta) \right] + J \right) t \right] \quad (10)$$

Averaging this over all molecular orientation produces (rather unusually) a reasonably neat analytical expression for the DEER/ESEEM signal:

$$\int_0^\pi \cos \left[\left(D \left[1 - 3 \cos^2(\theta) \right] + J \right) t \right] \sin \theta d\theta = \sqrt{\frac{\pi}{6Dt}} \left[\cos \left[(D+J)t \right] \text{FrC} \left[\sqrt{\frac{6Dt}{\pi}} \right] + \sin \left[(D+J)t \right] \text{FrS} \left[\sqrt{\frac{6Dt}{\pi}} \right] \right] \quad (11)$$

where the Fresnel functions are defined as:

$$\text{FrC}(x) = \int_0^x \cos(t^2) dt \quad \text{FrS}(x) = \int_0^x \sin(t^2) dt \quad (12)$$

EPR spectrometers cannot excite the entire width of a typical g -tensor anisotropy pattern (Figure 1, top right), and therefore some orientation selection effects are normally present in DEER data, particularly at high field.

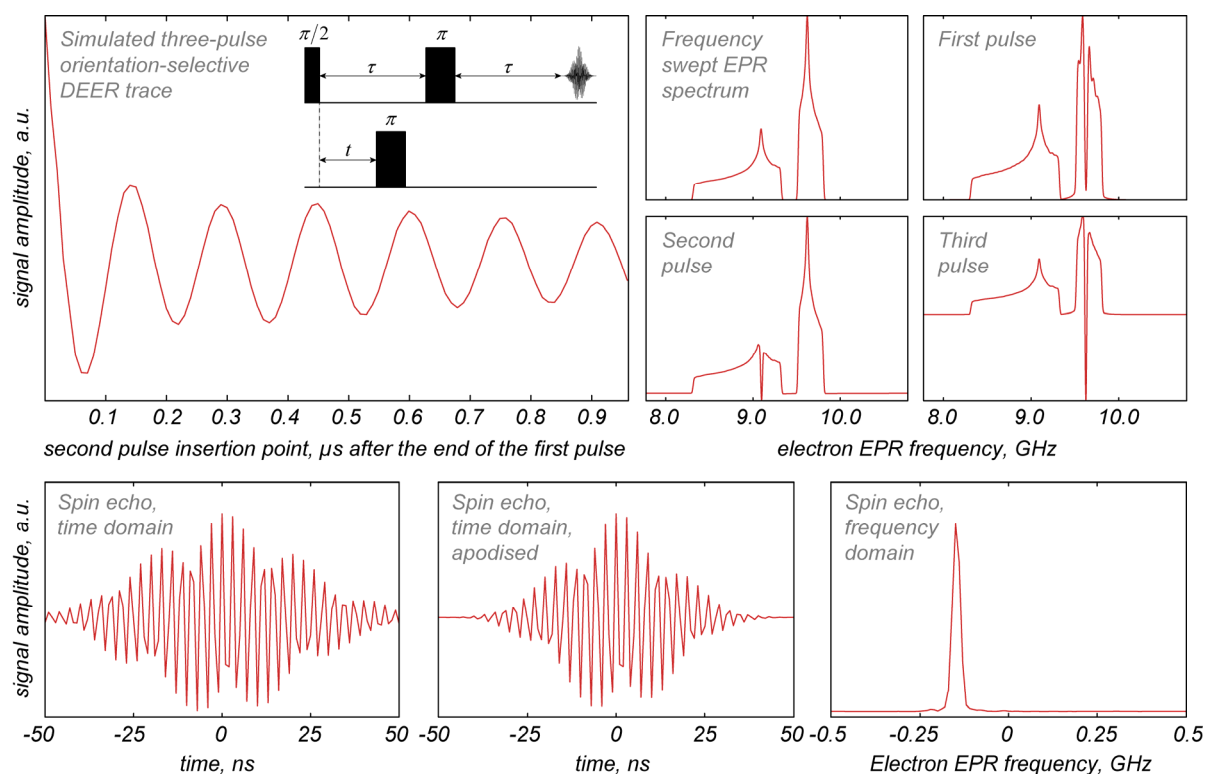


Figure 1. A three-pulse DEER trace and associated diagnostic information for a two-electron system at a magnetic induction of 0.34518 Tesla with the following parameters: first electron g -tensor eigenvalues [2.284, 2.123, 2.075], Euler angles [45°, 90°, 135°], second electron g -tensor eigenvalues [2.035, 2.013, 1.975], Euler angles [120°, 60°, 30°], inter-electron distance of 20 Angstrom along the X axis, pulse durations 20 ns, 40 ns and 40 ns, pulse amplitude 8 MHz for all three pulses, pulse frequencies 9.623 GHz, 9.092 GHz and 9.623 GHz, 1 μ s gap between the first and the third pulse, 100 steps in the position of the second pulse, 100 points in the echo sampling window.

Because Equation (11) is analytical, it is quite easy to extract distances, and even distance distributions (using a regularisation procedure) from DEER data – thousands of papers exist on this subject.