Magnetic Resonance Imaging: Advanced Concepts
CHEM 6154 – Nuclear Magnetic Resonance

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February 26, 2020
In this lecture, we will:

▶ Look at image reconstruction in more detail;
▶ Explore what pulse sequences are needed for cross-section images (tomography);
▶ Introduce some real-world MR imaging pulse sequences.

At the end, you will

▶ Understand the relationship between NMR data and MR images;
▶ Understand how selective pulses work;
▶ Understand the principle behind rapid imaging.
Principal component analysis (PCA) is an efficient tool to reduce the dimension of a data set consisting of a large number of interrelated variables while retaining most of the variations. It is achieved by transforming the data set to a new set of ordered variables according to their variances or their importance. This technique has three effects: it orthogonalizes the components of the input vectors so that uncorrelated with each other, it orders the resulting orthogonal components so that those with the largest variation come first, and eliminates those components contributing the least to the variation in the data set.

It should be noted that the input vectors should be normalized to have zero mean and unity variance before performing PCA, which is shown in Fig. 5. The normalization is a standard procedure. Details about PCA could be seen in Zhang, Wu, and Wei (2009).

5. BPNN

5.1. Structure

Neural networks are widely used in pattern classification since they do not need any information about the probability distribution and the a priori probabilities of different classes. A single-hidden-layer backpropagation neural network is adopted with sigmoid neurons in the hidden layer and linear neuron in the output layer.

The training vectors were presented to the NN, which is trained in batch mode. The network configuration is $N_I/N_H/N_O$, i.e., a two-layer network with $N_I$ input neurons, $N_H$ neurons in the hidden layer, and one output indicating the brain is normal or abnormal. The structure is depicted in Fig. 6. The experiment section will discuss the detailed method for determining the values of $N_I$ and $N_H$.

5.2. Training method

The basic backpropagation algorithm adjusts the weights in the steepest descent direction (negative of the gradient), the direction in which the performance function is decreasing most rapidly. Although the function decreases most rapidly along the negative of the gradient, this algorithm does not necessarily produce the fastest convergence (Zhang, Wu, Neggaz, et al., 2009). Mathematicians have already proven that conjugate gradient (CG) algorithms, searching along conjugate gradient directions, produces faster convergence than steepest descent directions (Gonzalez & Dorronsoro, 2008). Among CG algorithm group, scaled conjugate gradient (SCG) method is an excellent and powerful one. It was designed to avoid line search and can be regarded as the combination of model-trust region approach and the conjugate gradient approach (Kostopoulos & Grapsa, 2009). Thus, we utilize SCG to train the network.

6. Experiments and discussions

The experiments were carried out on the platform of P4 IBM with 3 GHz main frequency and 2G memory, running under Windows XP operating system. The algorithm was developed via the wavelet toolbox, the neural network toolbox, and the statistical toolbox.

Table 1

<table>
<thead>
<tr>
<th>Setting of training and test images.</th>
<th>Total no. of images</th>
<th>No. of images in training area (33)</th>
<th>No. of images in testing area (33)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Normal</td>
<td>66</td>
<td>9</td>
<td>24</td>
</tr>
<tr>
<td>Abnormal</td>
<td>9</td>
<td>24</td>
<td>9</td>
</tr>
</tbody>
</table>

Fig. 7. Sample of brain MRIs: (a) normal brain; (b) glioma; (c) meningioma; (d) Alzheimer; (e) Alzheimer plus visual agnosia; (f) Pick's disease; (g) sarcoma; (h) Huntington's disease.
4 | Outline

Acquisition of Image Data

Image Reconstruction

Slice Excitation and Selective Pulses

Selective Excitation

Gradient Echo Pulse Sequences

Recapitulation
5 | Sampling In $k$-Space

\[ S(\omega) = \int e^{-i\omega t} S(t) \, dt \]

\[ S(t) = \int e^{i\omega t} S(\omega) \, d\omega \]

$\omega = -\gamma G z$, hence $z = -(\gamma G)^{-1} \omega$

- reciprocal coordinate $k = -\gamma G t$
- \{k\} = $\frac{Hz}{T\text{ms}} = \frac{1}{m}$.

Sampling in time

In the presence of a field gradient $G$, each acquired data point $S(t)$ corresponds to a pixel of the image in reciprocal space at the location $k = -\gamma G t$. 
6 | Sampling in 2D

$^1\text{H}$

$\pi$

$t$

$G_x$

$G_z$

$k_z$

$k_x$
Nyquist Theorem

Image Size
is proportional to the resolution in k-space, i.e., the separation between neighboring points $\delta k$.

Image Resolution
is proportional to the size of the covered area in k-space, $\Delta k$.

\[
\delta x = \frac{2\pi}{\Delta k}
\]
\[
\Delta x = \frac{2\pi}{\delta k}
\]

Limits of MRI Resolution
Since $k = -\gamma G t$, the maximum achievable $\Delta k$ is determined by the available gradient strengths and the $T_2$ relaxation time.
Consider a sample consisting of identical spins (same chemical shift). Spin density $\rho(\mathbf{r})$. Signal contribution from each volume element $d\mathbf{r}$ in the presence of $G_x$:

$$dS(\mathbf{r}) = \rho(\mathbf{r}) \exp(-i \gamma G_x t_x \cdot x) d\mathbf{r}.$$ 

If we have gradients in two dimensions:

$$dS(\mathbf{r}) = \rho(\mathbf{r}) \exp(i k_x \cdot x) \exp(i k_y \cdot y) d\mathbf{r}$$

$$= \rho(\mathbf{r}) \exp(i \mathbf{k} \cdot \mathbf{r}) d\mathbf{r}$$
9 | Image Reconstruction

Fourier Transform
The MRI signal is the inverse Fourier transform of the weighted spin density:

\[ S(k) = \frac{1}{2\pi} \int \rho(r) \exp(i k \cdot r) \, dr \]

The image is therefore obtained by computing the Fourier Transform of the MRI signal:

\[ \hat{S}(r) = \int S(k) \exp(-i k \cdot r) \, dk. \]
10 | An Example

Magnitude Image 128 × 128

FID 128 × 128

Magnitude and Phase

k_x

k_y
11 | Phase Information Is Crucial!

Image Autocorrelation Function

FID

Magnitude Only
12 | Only Positive $k$-Values

Magnitude Image

FID

$k_x$, $k_y$
13 | Larger $k$-Space Steps

Magnitude Image

FID

FOV
14 | Fewer $k$-Space Steps

Magnitude Image

FID

$k_x$

$k_y$
15 | Even Fewer $k$-Space Steps

Magnitude Image

FID

$k_y$

$k_x$
16 | Ridiculously Few $k$-Space Steps

Magnitude Image

FID
17 | Imaging in 3 Dimensions

3D Imaging

- Hard pulse excitation
- entire volume excited in each transient
- time consuming: $n \times m \times l$-image requires $n \times m$ transients
- total acquisition time $\sim n \times m \times T_1$.

Slice Selection

- Frequency-selective excitation pulse of bandwidth $\Delta \omega$
- excitation slice bandwidth $G_z \Delta \omega$
- only a small fraction of the sample is excited
- total acquisition time $\sim n \times T_1$. 
All RF Pulses are Frequency-Selective

Source: Keeler
19 | Excitation Profiles

Rectangular \((\frac{\pi}{2})_x\) Pulse

(a) \(M_y\) vs \(\Omega / \omega_1\)

(b) \(M_x\) vs \(\Omega / \omega_1\)

Rectangular \((\pi)_x\) Pulse

(a) \(M_z\) vs \(\Omega / \omega_1\)

(b) \(M_z\) vs \(\Omega / \omega_1\)
NMR Response to RF is Nonlinear!

\[ \ddot{g}(t) + \delta \dot{g}(t) + \omega_0^2 g(t) = f(t) \]

Block Equations (rotating frame)

\[
\frac{\partial}{\partial t} \begin{pmatrix} M'_x \\ M'_y \\ M'_z \end{pmatrix} = \begin{pmatrix} -\frac{1}{T_2} & \Omega & 0 \\ -\Omega & -\frac{1}{T_2} & \omega_{rf} \\ 0 & -\omega_{rf} & -\frac{1}{T_1} \end{pmatrix} \begin{pmatrix} M'_x \\ M'_y \\ M'_z \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \frac{M_0}{T_1} \end{pmatrix}
\]

Bloch Equations

\[ \begin{pmatrix} B_1(t) \\ \Omega(t) \\ \phi(t) \end{pmatrix} \quad \text{Input} \quad \text{Spin System} \quad \begin{pmatrix} M_x(t) \\ M_y(t) \\ M_z(t) \end{pmatrix} \quad \text{Response} \]
NMR Response to RF is Nonlinear!

\[ \ddot{g}(t) + \delta \dot{g}(t) + \omega_0^2 g(t) = f(t) \]

Input \[ f(t) \] \hspace{1cm} Linear System \hspace{1cm} Response \[ g(t) \]

\[ \dot{g}(\omega)(-\omega^2 + i\omega\delta + \omega_0^2) = \hat{f}(\omega) \]

Bloch Equations (rotating frame)

\[
\frac{\partial}{\partial t} \begin{pmatrix} M'_x \\ M'_y \\ M'_z \end{pmatrix} = \begin{pmatrix} -\frac{1}{T_2} & \Omega & 0 \\ -\Omega & -\frac{1}{T_2} & \omega_{rf} \\ 0 & 0 & -\frac{1}{T_1} \end{pmatrix} \begin{pmatrix} M'_x \\ M'_y \\ M'_z \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \frac{M_0}{T_1} \end{pmatrix}
\]

Bloch Equations
**22 | Small Flip Angle Limit**

**Bloch Equations (rotating frame, no relaxation)**

\[
\frac{\partial}{\partial t} \begin{pmatrix}
M'_x \\
M'_y \\
M'_z
\end{pmatrix} = \begin{pmatrix}
0 & \Omega & 0 \\
-\Omega & 0 & -\gamma B_1 \\
0 & \gamma B_1 & 0
\end{pmatrix} \begin{pmatrix}
M'_x \\
M'_y \\
M'_z
\end{pmatrix}
\]

For small flip angles, we can set \( M'_z \approx M_0 \) and \( \dot{M}'_z = 0 \):

\[
\dot{M}_x(t) = \Omega M_y(t)
\]
\[
\dot{M}_y(t) = -\Omega M_x(t) - \gamma B_1(t) M_0.
\]

Elimination of \( M_y(t) \) yields the small flip-angle ODE:

\[
\ddot{M}_x(t) + \Omega^2 M_x(t) = -\Omega \gamma B_1(t) M_0
\]

- **Linear ODE**
- **Driving Term**
23 | Small Flip Angle Response is Linear

Fourier Transform of Response

\[ \dot{M}_x(t) + \Omega^2 M_x(t) = -\Omega \gamma B_1(t) M_0 \]
\[ \downarrow \mathcal{F} \]
\[ (-\omega^2 + \Omega^2) \hat{M}_x(\omega) = -\Omega M_0 \gamma \hat{B}_1(\omega) \]
\[ \hat{M}_x(\omega) = \frac{\Omega}{\omega^2 - \Omega^2} M_0 \gamma \hat{B}_1(\omega). \]
Consider a pulse of duration $\tau$ with an arbitrary envelope $B_+(t) = B_{1x} + i B_{1y}$ in the presence of a constant field gradient: $\mathbf{B}_+$. We introduce $M_+ = M_x + i M_y$

Equation of motion:

$$\dot{M}_+(t) + i \gamma \mathbf{G} \cdot \mathbf{r} M_+(t) = i \gamma M_0 B_+(t)$$

ODE of the form $\dot{f}(t) + P f(t) = Q(t)$. Solution by the \textit{integrating factor} technique.

\footnote{Everything here in the rotating frame.}
Small Flip Angle Response

\[ M_+(r, \tau) = i \gamma M_0 e^{-i \gamma G \cdot r \tau} \int_{-\infty}^{\tau} B_+(t) e^{i \gamma G \cdot r \tau} dt \]

Phase factor

\[ \mathcal{F}^{-1}[B_+(t)] \]

\[ B_+ = 0 \]

\[ t \]

\[ t_d \]

\[ B_+ = 0 \]

\[ M_+(r, \frac{t_d}{2}) = i \gamma M_0 e^{-i \gamma G \cdot r \tau d/2} \int_{-\infty}^{\infty} B_+(t) e^{i \gamma G \cdot r \tau} dt \]
26 | Sinc-shaped Pulses

\[ \text{sinc}(\frac{2\pi t}{t_0}) \]

Fourier transform:

\[ \frac{\omega t_0}{2\pi} \]
Selective Slice Excitation by $3^\circ$ Sinc Pulse

\[ \gamma \mathbf{G} \cdot \mathbf{r} t_0 / 2\pi \]

\[ M_y \]

\[ M_x \]
Selective Excitation Requires Refocusing

\[ M_+(r, \tau) = i \gamma M_0 e^{-i \gamma G \cdot r t_d / 2} \int_{-\infty}^{\infty} B_+(t) e^{i \gamma G \cdot r t} \, dt \]
29 | 3° Sinc Pulse with Refocusing

\[ M_x = \gamma G \cdot r \cdot \frac{t_0}{2\pi} \]

\[ M_y \]
30 | 90° Sinc Pulse

\[ T \]

- \( H^1 \)
- \( G \)
- \( 0.5 t_d \)
- \( 0.4965 t_d \)

\[ T \]
31 | Gradient Echo Sequence

$^1\text{H}$

$G_{\text{slice}}$

$G_{\text{phase}}$

$G_{\text{read}}$

ADC
32 | Multiple Gradient Echoes

\[ T_E \]

1H \hspace{1cm} t

G_{slice} \hspace{1cm} G_{phase} \hspace{1cm} G_{read} \hspace{1cm} ADC

\[ T_s \quad T_s \quad T_s \]
33 | Echo Planar Imaging

![Diagram of Echo Planar Imaging](image)

- $^1H$
- $G_{\text{slice}}$
- $G_{\text{phase}}$
- $G_{\text{read}}$
- ADC

Parameters:
- $T_E$
- $T_s$
Take-home messages from today:

- The FID data is the Fourier transform of the MR image;
- Each signal data point corresponds to a pixel in $k$-space;
- Slice selection is based on the combination of field gradients and selective RF pulses;
- The response of the spin system to RF pulses is non-linear in general;
- Low-intensity RF pulses have narrow bandwidth;
- Gradient echoes allow rapid acquisition of MR images.