

Lecture 8: Pulsed field gradients and k -space

To make NMR signal acquisition spatially selective, some property of spin dynamics must be made location-dependent. In magnetic resonance imaging (MRI), this property is Larmor frequency. MRI instruments have an extra set of coils (Figure 1) that make the magnetic field location-dependent:

$$\omega(\mathbf{r}) = -\gamma B(\mathbf{r}), \quad \mathbf{r} = [x \ y \ z] \quad (1)$$

This makes spins precess at different frequencies in different parts of the sample. This lecture is about the details of this technique and its applications in MRI.

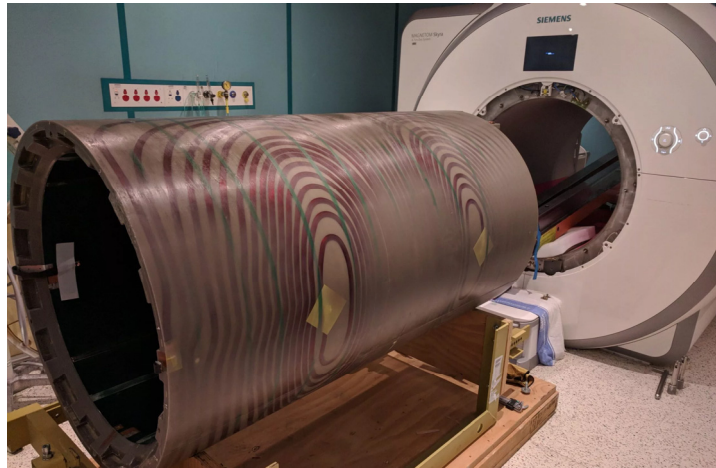


Figure 1. Gradient coils of a Siemens MRI scanner. Source: DirectMed Imaging Ltd.

1. Pulsed field gradients

Consider a magnetic field, still directed along the Z axis, whose amplitude (but not direction) depends linearly on the three spatial coordinates:

$$B(\mathbf{r}) = B_0 + g_x x + g_y y + g_z z \quad (2)$$

Mathematically, the gradient of this magnetic field is:

$$\nabla B = \left[\frac{\partial B}{\partial x} \quad \frac{\partial B}{\partial y} \quad \frac{\partial B}{\partial z} \right] = [g_x \quad g_y \quad g_z] \quad (3)$$

This vector contains the three slopes of the location dependence in Eq (2). Each slope depends linearly on the electric current that the instrument sends through the coils shown in Figure 1. Such currents are often pulsed, hence the name: **pulsed field gradients**.

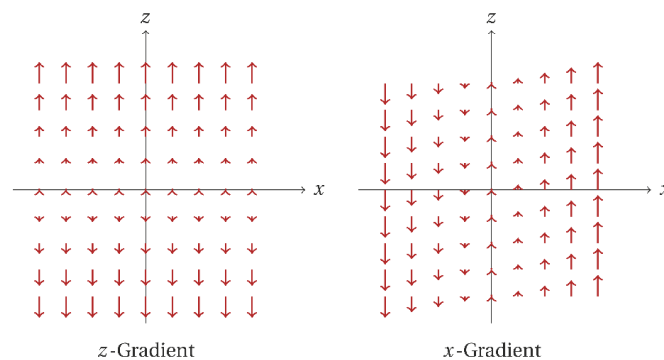


Figure 2. Quiver plots of (left) a Z-gradient of a Z magnetic field (right) an X-gradient of a Z magnetic field.

Schematics of the magnetic field profiles generated by Z and X gradient coils of an MRI instrument are given in Figure 2. It is important to remember that the magnetic field continues to point in the Z direction: X gradient corresponds to the variation of the Z *magnetic field* along the X axis.

2. Magnetic resonance imaging in one dimension

Consider a standard 5 mm NMR tube placed into a uniform magnetic field and containing a substance that has a single sharp NMR line (Figure 3). Even if substance concentration varied across the tube, the instrument would still record a single-line spectrum.

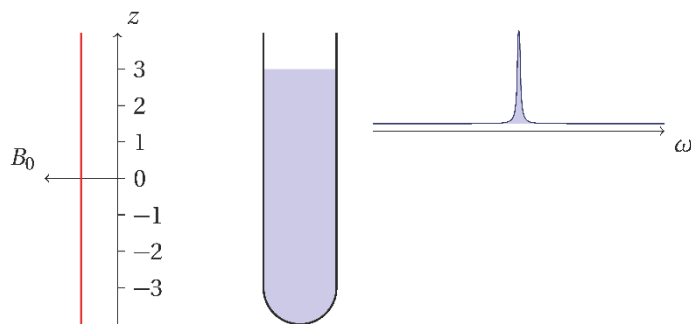


Figure 3. A schematic drawing of the NMR spectrum of a single-line sample in a uniform magnetic field.

However, if a magnetic field gradient is turned on, different horizontal *slices* of the sample would have different Larmor frequencies (Figure 4). After the Fourier transform, NMR spectra of individual slices would combine to form an image of the sample, with the line intensity distribution reflecting the concentration distribution in the sample.

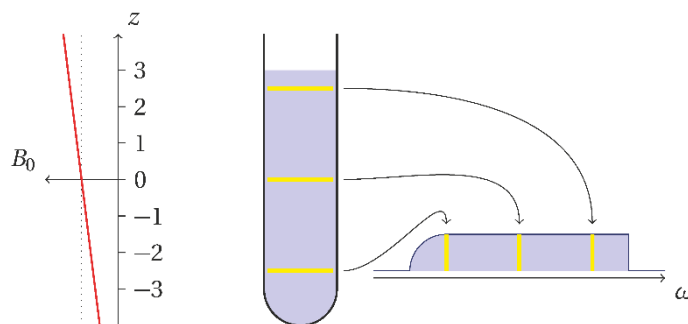


Figure 4. A schematic drawing of the NMR spectrum of a single-line sample in a linearly varying magnetic field.

Thus, by applying a magnetic field gradient, we could record an image of the sample. Let us now proceed to develop this technique further.

3. Gradient spirals

Typical gradients used in MRI (30-50 mT/m) do not significantly alter the B_0 field (several Tesla) and therefore have little influence on the equilibrium Z-magnetisation. However, such gradients do significantly alter the transverse magnetisation precession rate because $\Delta B_0 = 50$ mT corresponds to a change in proton Larmor frequency by about 2 MHz. The result is illustrated in Figure 5 – a magnetic field gradient winds the transverse magnetisation up into a spiral.

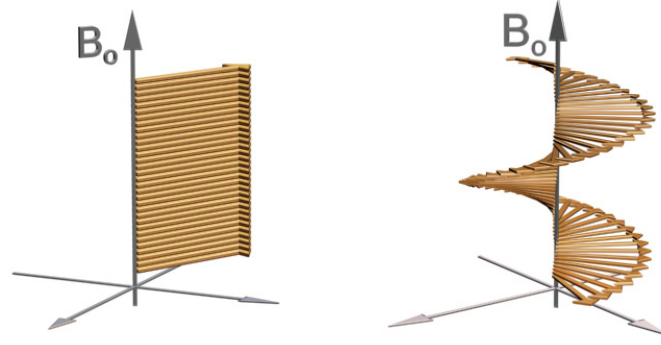


Figure 5. A schematic illustration of the transverse magnetisation spiral resulting from spin precession in a magnetic field that changes linearly across the sample. **Left:** uniform magnetic field. **Right:** Z-gradient of the magnetic field.

The tightness of the spiral, *i.e.* the turn angle per unit length of the sample, is called the ***k*-value**; the term comes from the expression for the magnetisation vector as a function of coordinate:

$$\begin{bmatrix} \cos(kz) & \sin(kz) \end{bmatrix} \Rightarrow \cos(kz) + i \sin(kz) = e^{ikz} \quad (4)$$

where the two-dimensional vector is represented by a complex number because it is computationally convenient. The *k*-value is determined by the amplitude and the duration of the gradient:

$$k = -\gamma g_z t_G \quad (5)$$

where γ is the magnetogyric ratio of the working nuclei (rad/s·T), g_z is the magnetic field gradient (T/m), and t_G is the time that the gradient was active for (seconds). It follows that *k* has units of rad/m, which makes sense given the definition in Eq (4).

4. One-dimensional *k*-space

Consider now a mathematical formulation of the problem introduced in Section 2: a sample with proton probability density distribution (“image”) $p(z)$ within some finite interval of the Z axis. When its magnetisation is made transverse and the gradient is turned on, the overall magnetic moment seen by the instrument is an integral over the sample:

$$\mu(t) \propto \int_{-\infty}^{\infty} p(z) e^{-i\gamma g_z t z} dz \quad (6)$$

where we can recognise a Fourier transform. The image $p(z)$ may therefore be recovered from the magnetisation trajectory $\mu(t)$ by an inverse Fourier transform, in agreement with Section 2.

Every instrument has different values of g_z and therefore different timing settings. To make the treatment general, the notation is adapted to use Equation (5):

$$\mu(k) \propto \int_{-\infty}^{\infty} p(z) e^{ikz} dz \Rightarrow p(z) \propto \int_{-\infty}^{\infty} \mu(k) e^{-ikz} dk \quad (7)$$

This Fourier transform pair creates a map between the linear space of all images (called ***image space***) and the linear space of all gradient-driven magnetisation trajectories (called ***k-space***). This equation is the foundation of magnetic resonance imaging.