

Week 1 workshop exercises

1. Open the brackets and express as a single complex number:

$$(2+3i)+(4-5i) \quad (5+3i)(3-i) \quad (1-3i)^2 \quad (1-3i)(1+3i)$$

2. For $z = 3 - 2i$:

(a) find z^* ;

(b) find zz^* , and hence $|z|$;

3. Perform the division and express as a single complex number:

$$\frac{1-i}{1+i} \quad \frac{1}{5+3i} \quad \frac{1}{5} - \frac{3-4i}{3+4i}$$

4. Plot as a points on the complex plane, and find the amplitudes and the phases:

$$2i \quad -3 \quad 1-i \quad 1/i$$

5. Given the complex function $f(x) = 3x^2 + (1+2i)x + 2(i-1)$,

(a) express it in the form $f(x) = g(x) + ih(x)$, where $g(x)$ and $h(x)$ are real functions;

(b) solve $g(x) = 0$ and $h(x) = 0$, and hence $f(x) = 0$;

6. Express in the exponential form $z = |z|e^{i\varphi}$ and use that form to find the conjugate and the inverse:

$$1-i \quad \sqrt{3}+i \quad 2i \quad -3$$

7. Derive the reciprocal relations in Equation (11) of the lecture notes from Equation (10). Proceed by writing Euler's formula for $e^{i\varphi}$ and $e^{-i\varphi}$, and then adding or subtracting those expressions.

8*. Quantum mechanical wavefunctions for a rotating methyl group are $\psi_n(\varphi) = Ce^{in\varphi}$, where n is an integer, and C is a positive real number called a *normalisation constant*.

(a) Find the normalisation constant for which

$$\int_0^{2\pi} \psi_n^*(\varphi)\psi_n(\varphi) d\varphi = 1$$

(b) Demonstrate that

$$\int_0^{2\pi} \psi_m^*(\varphi)\psi_n(\varphi) d\varphi = 0 \quad \text{if } m \neq n$$

9*. Use the formulae you have derived in P7 to convert trigonometric functions into sums and differences of exponentials, then take the integrals:

$$\int_0^{\infty} e^{-x} \cos(2x) dx \quad \int_0^{\infty} e^{-3x} \sin(x) dx$$