Limits P1

(a) $a_n = 1 + 3(n-1) = 3n - 2$

(b) $a_n = 3^{n-1}$ (c) $a_n = \left(-\frac{1}{5}\right)^{n-1}$

Limits P2

(a) $u_0 = 1$; $u_1 = 1$; $u_2 = 2$; $u_3 = 4$; $u_4 = 7$; $u_5 = 11$.

(b) $u_0 = 1$; $u_1 = 1$; $u_2 = 3$; $u_3 = 7$; $u_4 = 17$; $u_5 = 41$.

Limits P3

(a) $\lim_{r \to 0} \frac{1}{3^r} = 1$; $\lim_{r \to \infty} \frac{1}{3^r} = 0$.

(b) $\lim_{r \to 0} 2^r = 1$; $\lim_{r \to \infty} 2^r = \infty$.

(c) $\lim_{r \to 0} \frac{1}{r+2} = \frac{1}{2}$; $\lim_{r \to \infty} \frac{1}{r+2} = 0$.

(d) $\lim_{r \to 0} \frac{r}{r+2} = 0$; $\lim_{r \to \infty} \frac{r}{r+2} = 1$.

(e) $\lim_{r \to 0} \frac{r}{r^2+r+1} = 0$; $\lim_{r \to \infty} \frac{r}{r^2+r+1} = 0$.

(f) $\lim_{r \to 0} \frac{3r^2+3r+1}{5r^2-6r-1} = -1$; $\lim_{r \to \infty} \frac{3r^2+3r+1}{5r^2-6r-1} = \frac{3}{5}$.

Limits P4

(a) $\lim_{x \to 0} x^2 e^{-x} = 0$ $\lim_{x \to \infty} x^2 e^{-x} = 0$

(b) $\lim_{x \to 0} \cos(2x) e^{-x} = 1$ $\lim_{x \to \infty} \cos(2x) e^{-x} = 0$

Limits P5

$$C_v = 3R\left(\frac{h\nu}{kT}\right)^2 \frac{e^{h\nu/kT}}{\left(e^{h\nu/kT}-1\right)^2} = 3R\,x^2\frac{e^x}{(e^x-1)^2}$$

$$\lim_{T\to 0} C_v(T) = \lim_{x\to\infty} C_v(x)$$

$$\lim_{x\to\infty} 3R\frac{x^2e^x}{(e^x-1)^2} = \lim_{x\to\infty} 3R\frac{x^2e^x}{e^{2x}} = \lim_{x\to\infty} 3R\,x^2e^{-x} = 0$$

Limits P6

$$\lim_{r\to 0} N\left(\frac{r}{a_0}\right)^2 \exp\left(-\frac{r}{a_0}\right) = 0$$

$$\lim_{r\to\infty} N\left(\frac{r}{a_0}\right)^2 \exp\left(-r/a_0\right) = 0$$

$\Rightarrow$ no electron probability around the nucleus (too far for 3s), no electron probability at infinity (reasonable).

WBF P1

$y = x^2$ is equal to its limits everywhere.

WBF P2

$f(x) > 0$ and continuous
$\sqrt{x}$ known continuous
$\Rightarrow$ superposition continuous

WBF P3

$\tan x = \frac{\sin x}{\cos x}$, problems where $\cos x = 0 \Rightarrow$

$\Rightarrow x = \frac{\pi}{2} + \pi n, \quad n \in \mathbb{Z}$

WBF P4

Function must be equal to its limits:

$$\lim_{x\to 2}\left(\frac{x^2-4}{x-2}\right) = \lim_{x\to 2}\left(\frac{(x-2)(x+2)}{x-2}\right) =$$

$$= \lim_{x\to 2}(x+2) = 4 \quad \Rightarrow \quad a = 4.$$

WBF P5

Take any discontinuous function and subtract it from itself. 0 is continuous. Other examples possible, of course.

WBF P6

(a) $y = \frac{x^2}{x-2}$ - discontinuity & singularity at $x=2$

(b) $y = \frac{1+x^3}{1+x}$ - not defined at $x=-1$, but can be fixed if set to the value of its limit:

$$\lim_{x\to -1}\left(\frac{1+x^3}{1+x}\right) = \lim_{x\to -1}(1-x+x^2) = 3$$

(c) $y = \frac{x}{|x|}$ - not defined at $x=0$, has a jump discontinuity there

(d) $y = \exp\left(\frac{1}{x+1}\right)$ - singularity & discontinuity at $x=-1$.