

### Week 3 workshop exercises

1. Differentiate using limits: (a)  $2x^2 + 3x + 4$  (b)  $x^4$
2. Differentiate using the table of simple derivatives:  
(a)  $x^3$  (b)  $x^{5/4}$  (c)  $x^{1/3}$  (d)  $1 - 2x + 3x^2 - 4x^3 + 5 \sin x - 6 \cos x + 7e^x - 8 \ln x$
3. An approximate equation of state for a non-ideal gas at low pressure is:

$$pV = nRT \left( 1 - \frac{nB}{V} \right)$$

where  $B$  is an experimentally determined constant. Solve for  $P$  and find  $\partial p / \partial V$ .

4. Differentiate using product, quotient, and chain rules:

(a)  $(1 - 4x^2) \cos x$  (b)  $(2 + 3x)e^x$  (c)  $e^x \cos x$  (d)  $x \ln x$  (e)  $(1 + x)^5$

(f)  $\sqrt{2 + x^2}$  (g)  $e^{-2x}$  (ö)  $\cos(2x^2 - 3x + 1)$  (ξ)  $e^{\sin x}$  (ϋ)  $\ln[(2 + x)/(3 - x)]$

(ю)  $\ln(\sin 2x + \sin^2 x)$  (ж)  $\tan(4x) \cos^2(2x)$  (æ)  $x^2 e^{2x^2 + 3}$

5. Find  $\partial z / \partial x$  and  $\partial z / \partial y$  for:

(a)  $z = 2x^2 - y^2$  (b)  $z = \sin(x^2 - y^2)$

6. Find all non-zero partial derivatives of:

(a)  $z = x^2 - 3x^2y + 4xy^2$  (b)  $u = 3x^2 + y^2 + 2xy^2$

7. Find all first and second partial derivatives of  $z = 2x^2y + \cos(x + y)$ .

8. Show that  $f''_{xy} = f''_{yx}$  for: (a)  $f = x^3 - 3x^2y + y^3$  (b)  $f = x^2 \cos(y - x)$

9. Show that  $f'''_{xyz} = f'''_{yxz} = f'''_{zxy}$  for  $f = \cos(x + 2y + 3z)$ .

- 10\*. Show that the solutions of the one-dimensional wave equation

$$\frac{\partial^2 \varphi}{\partial x^2} = \frac{1}{c^2} \frac{\partial^2 \varphi}{\partial t^2}$$

where  $c$  is the speed of light, have the following general form:

$$\varphi = f(x - ct) + g(x + ct)$$

where  $f$  and  $g$  are arbitrary well-behaved functions.