CHEM1047 – Week 5 problem set

Make sure that the trigonometric functions in your calculator are set to radians, not degrees.

1. Using Newton-Raphson method, find all roots of the following equations within the indicated intervals to four decimal places:

   (a) $1-x=2\sin x$, all real x (b) $x^2=2+\ln x,\ 0<x\le 2$

2. Using Newton-Raphson method, find all stationary points of the following functions within the indicated intervals to four decimal places, and determine their type:

   (a) $f(x)=e^{-x}\ln(x),\ x>1$ (b) $f(x)=\cos\left(x^2+3\right),\ \pi/2<x\le\pi$

3. Calculate differentials of the following functions:

   (a) $y=2x$ (b) $y=3x^2+2x+1$ (c) $y=\sin x$

4. The volume of a sphere of radius r is $V(r)=4\pi r^3/3$. Calculate the differential dV and explain its geometric meaning.

5. Find the total differentials of the following functions:

   (a) $f(x,y)=x^3y^2+\ln y$ (b) $f(r,\theta,\varphi)=r\sin\theta\sin\varphi$

6. Test the following differentials for exactness:

   (a) $(4x+3y)\,dx+(3x+8y)\,dy$ (b) $y\cos(x)\,dx+\sin(x)\,dy$

7. The differential of the Gibbs free energy in chemical thermodynamics is

$$dG=-S(T,P)\,dT+V(T,P)\,dP.$$

   where S is entropy, V is volume, and both of these quantities depend on pressure P and temperature T. Show that the following relation:

$$\frac{\partial S}{\partial P}=\frac{\partial V}{\partial T}$$

   is equivalent to demanding that the differential dG be exact.