Week 6 workshop exercises

1. Find general solutions for the following differential equations:

(a) $\frac{dy}{dx}=\frac{3x^2}{y}$ (b) $\frac{dy}{dx}=4xy^2$ (c) $\frac{dx}{dt}=2(x+1)t$

2. Solve the initial value problems for the following ODEs:

(a) $\frac{dy}{dx}=\frac{y+1}{x-3},\quad y(0)=1$ (b) $\frac{dy}{dx}=\frac{x^2-1}{2y+1},\quad y(0)=-1$

Note: in part (b), the quadratic equation for y may be left unsolved.

3. Find the interval $\tau_{1/2}$ in which the amount of reactant A in a first order decay process

$$A(t)=A(0)e^{-kt}$$

is reduced by a factor of 2.

4. The following function was proposed as an approximation to $\cos(x)$ around $x=0$:

$$f(x)=\frac{1-\frac{115x^2}{252}+\frac{313x^4}{15120}}{1+\frac{11x^2}{252}+\frac{13x^4}{15120}}$$

Using a calculator, inspect the performance of this approximation (a) at the origin; (b) in the interval between $-\pi/2$ and $+\pi/2$; (c) around $x=2\pi$; (d) at infinity. The easiest way to proceed is to pick a grid of x points, and to compare the values of the function and the approximation at those points.

5. Using the substitution $x=kT/\Delta E$, demonstrate (by calculating the function for a few values of x, or by carefully plotting both functions) that the following expression

$$Q(T)=\frac{1}{2}+\frac{kT}{\Delta E}+\frac{\Delta E}{12kT}$$

can be an excellent approximation to the partition function of the ensemble of harmonic oscillators

$$Q(T)=\frac{1}{1-e^{-\Delta E/kT}}$$

Use your calculator to estimate the interval of $kT/\Delta E$ for which the approximation is good (you would first need to decide what is to be considered "good").

6. Show that a differential equation of the form

$$\frac{dy}{dx}=f(ax+by+c)$$

where a,b,c are constants, is reduced to a separable form by the substitution $u=ax+by$. Proceed by calculating the differential du and using the result to eliminate dy from the numerator.