CHEM1047 - Week 9 - Workshop problem set

1. Evaluate the following integrals:

   (a) $\int x^7 dx$ (b) $\int 2y^5 dy$ (c) $\int x^{1.3} dx$ (d) $\int\left(\frac{4}{g^5}-\frac{3}{g^2}\right)dg$

   (e) $\int\sqrt{x^7}\,dx$ (f) $\int\cos(4t)\,dt$ (g) $\int\left(\varphi^4-\sin 2\varphi\right)d\varphi$

   (h) $\int\left(\frac{5}{z}+e^{4z}\right)dz$ (i) $\int\xi(\xi+a)(\xi+b)\,d\xi$

2. Diffusion processes at electrodes mean that, once the electrode is polarised, the current decays with time according to Cottrell's equation:

$$I(t)=nFAc\sqrt{\frac{D}{\pi t}}$$

   where n is the number of electrons transferred in the elementary reaction step, F is the charge of one mole of electrons, A is the electrode area, c is the concentration of the active substance, and D is the diffusion coefficient. The total charge is defined as the integral of the current. Determine the total charge $Q(t)$ that has flown through the electrode as a function of time.

3. Evaluate the following integrals:

   (a) $\int_0^{\pi/2}\cos^2(3x)\,dx$ (b) $\int_0^{\pi/2}\sin(2\varphi)\cos(2\varphi)\,d\varphi$ (c) $\int_0^{\pi}\sin(\alpha)\cos(2\alpha)\,d\alpha$

4. The wavefunction for an electron in a one-dimensional box of length L is:

$$\psi_k(x)=\sqrt{\frac{2}{L}}\sin\left(\frac{\pi kx}{L}\right)$$

   where k is a positive integer giving the number of the energy level. Demonstrate that wavefunctions corresponding to different energy levels are orthogonal:

$$\int_0^L\psi_n^*(x)\psi_m(x)\,dx=\begin{cases}1 & \text{if } n=m\\ 0 & \text{if } n\neq m\end{cases}$$

5. Evaluate the following integrals:

   (a) $\int\frac{dx}{x^2-9}$ (b) $\int\frac{dx}{x^2+4}$ (c) $\int\frac{x+1}{2x+1}dx$