

Problem 1

`Unprotect[C];`

$$A = \begin{pmatrix} 1 & -2 & 3 \\ 0 & 3 & 4 \end{pmatrix}; \quad B = \begin{pmatrix} 0 & 1 & -4 \\ 2 & -3 & 0 \end{pmatrix}; \quad C = \begin{pmatrix} -5 & 3 \\ 4 & -1 \\ 2 & -1 \end{pmatrix}; \quad M = \begin{pmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & -1 \end{pmatrix};$$

$$P = \begin{pmatrix} 1 & -2 \\ 0 & 4 \end{pmatrix}; \quad Q = \begin{pmatrix} 3 & 0 \\ 0 & 1 \end{pmatrix}; \quad a = \begin{pmatrix} 0 \\ -3 \\ 1 \end{pmatrix}; \quad b = (2 \ 5 \ -2);$$

`Det[A]`

`...` **Det:** Argument {{1, -2, 3}, {0, 3, 4}} at position 1 is not a non-empty square matrix.

`Det[{{1, -2, 3}, {0, 3, 4}}]`

`Tr[A]`

4

`Det[M]`

-6

`Tr[M]`

4

`Det[P]`

4

`Tr[P]`

5

`Transpose[A] // MatrixForm`

$$\begin{pmatrix} 1 & 0 \\ -2 & 3 \\ 3 & 4 \end{pmatrix}$$

`Transpose[C] // MatrixForm`

$$\begin{pmatrix} -5 & 4 & 2 \\ 3 & -1 & -1 \end{pmatrix}$$

`Transpose[M] // MatrixForm`

$$\begin{pmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

`Transpose[a] // MatrixForm`

$$(0 \ -3 \ 1)$$

`Transpose[b] // MatrixForm`

$$\begin{pmatrix} 2 \\ 5 \\ -2 \end{pmatrix}$$

a + Transpose[b] // MatrixForm

$$\begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix}$$

Transpose[a] + b // MatrixForm

$$(2 \ 2 \ -1)$$

A.B // MatrixForm

Dot: Tensors $\{\{1, -2, 3\}, \{0, 3, 4\}\}$ and $\{\{0, 1, -4\}, \{2, -3, 0\}\}$ have incompatible shapes.

$\{\{1, -2, 3\}, \{0, 3, 4\}\} \cdot \{\{0, 1, -4\}, \{2, -3, 0\}\}$

B.C // MatrixForm

$$\begin{pmatrix} -4 & 3 \\ -22 & 9 \end{pmatrix}$$

C.B // MatrixForm

$$\begin{pmatrix} 6 & -14 & 20 \\ -2 & 7 & -16 \\ -2 & 5 & -8 \end{pmatrix}$$

C.P // MatrixForm

$$\begin{pmatrix} -5 & 22 \\ 4 & -12 \\ 2 & -8 \end{pmatrix}$$

P.C // MatrixForm

Dot: Tensors $\{\{1, -2\}, \{0, 4\}\}$ and $\{\{-5, 3\}, \{4, -1\}, \{2, -1\}\}$ have incompatible shapes.

$\{\{1, -2\}, \{0, 4\}\} \cdot \{\{-5, 3\}, \{4, -1\}, \{2, -1\}\}$

M.M // MatrixForm

$$\begin{pmatrix} 9 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

P.Q // MatrixForm

$$\begin{pmatrix} 3 & -2 \\ 0 & 4 \end{pmatrix}$$

B.a // MatrixForm

$$\begin{pmatrix} -7 \\ 9 \end{pmatrix}$$

a.b // MatrixForm

$$\begin{pmatrix} 0 & 0 & 0 \\ -6 & -15 & 6 \\ 2 & 5 & -2 \end{pmatrix}$$


b.a // MatrixForm

$$(-17)$$

Transpose[a].Transpose[b] // MatrixForm

$$(-17)$$

C.a // MatrixForm

 **Dot:** Tensors $\{\{-5, 3\}, \{4, -1\}, \{2, -1\}\}$ and $\{\{0\}, \{-3\}, \{1\}\}$ have incompatible shapes.

$\{\{-5, 3\}, \{4, -1\}, \{2, -1\}\} \cdot \{\{0\}, \{-3\}, \{1\}\}$

Transpose[a].C // MatrixForm

$(-10 \ 2)$

Problem 2

$\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ // MatrixForm

$\begin{pmatrix} y \\ x \\ z \end{pmatrix}$

Problem 3

$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ // MatrixForm

$\begin{pmatrix} x \\ y \\ 0 \end{pmatrix}$

Problem 4

Eigenvalues $\left[\begin{pmatrix} 2 & 2 \\ 1 & 3 \end{pmatrix}\right]$

Eigenvectors $\left[\begin{pmatrix} 2 & 2 \\ 1 & 3 \end{pmatrix}\right]$ // Map[Normalize, #] &

{4, 1}

$\left\{\left\{\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right\}, \left\{-\frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}}\right\}\right\}$

Eigenvalues $\left[\begin{pmatrix} 3 & 1 \\ 1 & 3 \end{pmatrix}\right]$

Eigenvectors $\left[\begin{pmatrix} 3 & 1 \\ 1 & 3 \end{pmatrix}\right]$ // Map[Normalize, #] &

{4, 2}

$\left\{\left\{\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right\}, \left\{-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right\}\right\}$

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Eigenvalues[ $\begin{pmatrix} 1 & 2 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ ]
Eigenvectors[ $\begin{pmatrix} 1 & 2 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ ] // Map[Normalize, #] &
{3, -1, 1}
 $\left\{ \left\{ \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0 \right\}, \left\{ -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0 \right\}, \{0, 0, 1\} \right\}$ 

Eigenvalues[ $\begin{pmatrix} 3 & 1 \\ -1 & 3 \end{pmatrix}$ ]
Eigenvectors[ $\begin{pmatrix} 3 & 1 \\ -1 & 3 \end{pmatrix}$ ] // Map[Normalize, #] &
{3 + i, 3 - i}
 $\left\{ \left\{ -\frac{i}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right\}, \left\{ \frac{i}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right\} \right\}$ 

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Problem 5

```

A = Eigenvectors[ $\begin{pmatrix} 3 & 1 \\ 1 & 3 \end{pmatrix}$ ];
{A[[1]]}.Transpose[{A[[2]]}]
{{0}}

A = Eigenvectors[ $\begin{pmatrix} 1 & 2 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ ];
{A[[1]]}.Transpose[{A[[2]]}]
{A[[2]]}.Transpose[{A[[3]]}]
{A[[1]]}.Transpose[{A[[3]]}]
{{0}}
{{0}}
{{0}}

```

Problem 6

```

ConjugateTranspose[ $\begin{pmatrix} 1+i & 2-i \\ 3+i & -i \end{pmatrix}$ ] // MatrixForm
 $\begin{pmatrix} 1-i & 3-i \\ 2+i & i \end{pmatrix}$ 

HermitianMatrixQ[ $\begin{pmatrix} 1+i & 2-i \\ 3+i & -i \end{pmatrix}$ ]
False

ConjugateTranspose[ $\begin{pmatrix} 2 & i \\ -i & 2 \end{pmatrix}$ ] // MatrixForm
 $\begin{pmatrix} 2 & i \\ -i & 2 \end{pmatrix}$ 

```

```
HermitianMatrixQ[ $\begin{pmatrix} 2 & i \\ -i & 2 \end{pmatrix}$ ]
```

```
True
```

```
ConjugateTranspose[ $\begin{pmatrix} 0 & -i & 0 \\ i & 0 & i \\ 0 & -i & 0 \end{pmatrix}$ ] // MatrixForm
```

```
 $\begin{pmatrix} 0 & -i & 0 \\ i & 0 & i \\ 0 & -i & 0 \end{pmatrix}$ 
```

```
HermitianMatrixQ[ $\begin{pmatrix} 0 & -i & 0 \\ i & 0 & i \\ 0 & -i & 0 \end{pmatrix}$ ]
```

```
True
```