Week 22 class problems

1. Evaluate the integral:

$$\int_L \left[(xy)\,dx + (2y)\,dy \right]$$

on the line $y = 2x$ from $x = 0$ to $x = 2$.

2. Evaluate the integral:

$$\int_L \left[\left(x^2 + 2y\right) dx + \left(y^2 + x\right) dy \right]$$

on a curve parameterised by $x = t$ and $y = t^2$ where t goes from 0 to 1.

3. Evaluate the integral:

$$\int_0^3 \int_1^2 \left(x^2 y + xy^2\right) dx dy$$

and show that the result does not depend on the order of integration.

4. Evaluate the integrals and sketch the region of integration:

$$\int_0^2 \int_x^{2x} \left(x^2 + y^2\right) dy dx \qquad \int_0^1 \int_0^{\sqrt{1-x^2}} \left(xy^2\right) dy dx$$

5. Show, using a sketch of the integration region, that:

$$\int_0^2 \int_{2y-4}^{(2-y)/2} x^3 dx dy = \int_0^1 \int_0^{2-2x} x^3 dy dx + \int_{-4}^0 \int_0^{(x+4)/2} x^3 dy dx$$

and evaluate the integral.

6. Find the total mass in the indicated volumes by integrating material density distributions:

   (a) $\rho = x^2 + y^2 + z^2$ V: the cube $0 \le x \le 1,\ 0 \le y \le 1,\ 0 \le z \le 1$

   (b) $\rho = xy^2 z^3$ V: the box $0 \le x \le a,\ 0 \le y \le b,\ 0 \le z \le c$

   (c) $\rho = x^2$ V: the region $1 - y \le x \le 1,\ 0 \le y \le 1,\ 0 \le z \le 2$

   (d) $\rho = \exp(-x - y - z)$ V: the region $0 \le x < \infty,\ 0 \le y < \infty,\ 0 \le z < \infty$