Problem 1

In[1]:=
$$\int_0^{\pi}\int_0^{R}\left(\text{Exp}[-r]\,\text{Cos}[\theta]^2\,\text{Sin}[\theta]\right)\,dr\,d\theta$$

Out[1]=
$$\frac{2}{3}-\frac{2\,e^{-R}}{3}$$

Problem 2

In[2]:=
$$\int_0^{1}\int_0^{\sqrt{1-x^2}}\left(x^2+2\,x\,y\right)\,dy\,dx$$

Out[2]=
$$\frac{4+\pi}{16}$$

In[3]:=
$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left(\text{Exp}\left[-2\sqrt{x^2+y^2}\,\right]\,\left(x^2+y^2\right)^3\right)\,dx\,dy$$

Out[3]=
$$\frac{315\,\pi}{8}$$

In[4]:=
$$\int_0^{\infty}\int_0^{\infty}\left(\text{Exp}\left[-\left(x^2+y^2\right)\right]\,x^2\right)\,dx\,dy$$

Out[4]=
$$\frac{\pi}{8}$$

Problem 3

In[16]:=
```
CoordinateTransform[ "Spherical" → "Cartesian", {1, θ, 0}] /. {θ → 0}
```

Out[16]=
```
{0, 0, 1}
```

In[7]:=
$$\text{CoordinateTransform}\left[\text{"Spherical"}\to\text{"Cartesian"},\left\{2,\frac{\pi}{2},\frac{\pi}{2}\right\}\right]$$

Out[7]=
```
{0, 2, 0}
```

In[8]:=
$$\text{CoordinateTransform}\left[\text{"Spherical"}\to\text{"Cartesian"},\left\{2,\frac{2\,\pi}{3},\frac{3\,\pi}{4}\right\}\right]$$

Out[8]=
$$\left\{-\sqrt{\frac{3}{2}},\sqrt{\frac{3}{2}},-1\right\}$$

Problem 4

In[17]:=
```
CoordinateTransform[ "Cartesian" → "Spherical", {1, 0, 0}]
```

Out[17]=
$$\left\{1,\frac{\pi}{2},0\right\}$$

In[18]:=
```
CoordinateTransform[ "Cartesian" → "Spherical", {0, 1, 0}]
```

Out[18]=
$$\left\{1,\frac{\pi}{2},\frac{\pi}{2}\right\}$$

In[23]:=
```
CoordinateTransform[ "Cartesian" → "Spherical", {1, 1, 0}]
```
Out[23]=
$$\left\{\sqrt{2}\,,\ \frac{\pi}{2},\ \frac{\pi}{4}\right\}$$

Problem 5

```
Sph = {x → r Sin[θ] Cos[φ], y → r Sin[θ] Sin[φ], z → r Cos[θ] };
```

In[27]:=
```
x^2 - y^2 /. Sph // Simplify
```
Out[27]=
$$r^2\,\text{Cos}[2\,\varphi]\,\text{Sin}[\theta]^2$$

In[28]:=
```
(x^2 + y^2)/z^2 /. Sph // Simplify
```
Out[28]=
$$\text{Tan}[\theta]^2$$

In[30]:=
```
2 z^2 - x^2 - y^2 /. Sph // Simplify
```
Out[30]=
$$\frac{1}{2}\,r^2\,\left(1 + 3\,\text{Cos}[2\,\theta]\right)$$

In[32]:=
```
Assuming[r > 0, (x^2 + y^2 + z^2)^(-1/2) /. Sph // Simplify]
```
Out[32]=
$$\frac{1}{r}$$

In[33]:=
```
Assuming[r > 0, ∂x (x^2 + y^2 + z^2)^(-1/2) /. Sph // Simplify]
```
Out[33]=
$$-\frac{\text{Cos}[\varphi]\,\text{Sin}[\theta]}{r^2}$$

Problem 6

In[35]:=
```
∫_0^a ∫_0^π ∫_0^(2 π) (r^2 r^2 Sin[θ]) 𝕕φ 𝕕θ 𝕕r
```
Out[35]=
$$\frac{4\,a^5\,\pi}{5}$$

In[36]:=
```
∫_0^a ∫_0^π ∫_0^(2 π) (Sin[θ]^2 Cos[φ]^2/r r^2 Sin[θ]) 𝕕φ 𝕕θ 𝕕r
```
Out[36]=
$$\frac{2\,a^2\,\pi}{3}$$

In[38]:=
```
∫_0^∞ ∫_0^π ∫_0^(2 π) (r^3 Exp[-r] r^2 Sin[θ]) 𝕕φ 𝕕θ 𝕕r
```
Out[38]=
$$480\,\pi$$

Problem 7

In[39]:=
$$\psi[r\_, \theta\_, \varphi\_] := \frac{1}{4\sqrt{2\pi a^3}}\left(2 - \frac{r}{a}\right) \text{Exp}\left[-\frac{r}{2a}\right];$$
$$\text{Assuming}\left[a > 0, \int_0^{\infty}\int_0^{\pi}\int_0^{2\pi}\left(\psi[r, \theta, \varphi]^2 r^2 \text{Sin}[\theta]\right) \text{d}\varphi\, \text{d}\theta\, \text{d}r\right]$$

Out[40]= 1