Week 23 workshop problems

1. Evaluate the integral:

$$\int_0^{\pi}\int_0^{R} e^{-r}\cos^2(\theta)\sin(\theta)\,dr\,d\theta$$

2. Transform to polar coordinates and evaluate:

$$\int_0^1\int_0^{\sqrt{1-x^2}}\left(x^2+2xy\right)dydx \qquad \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-2\sqrt{x^2+y^2}}\left(x^2+y^2\right)^3 dxdy \qquad \int_0^{\infty}\int_0^{\infty} e^{-\left(x^2+y^2\right)}x^2 dxdy$$

3. Find the Cartesian coordinates of the points whose spherical coordinates are:

$$(r,\theta,\varphi)=(1,0,0) \qquad (r,\theta,\varphi)=\left(2,\frac{\pi}{2},\frac{\pi}{2}\right) \qquad (r,\theta,\varphi)=\left(2,\frac{2\pi}{3},\frac{3\pi}{4}\right)$$

4. Find the spherical coordinates of the points whose Cartesian coordinates are:

$$(x,y,z)=(1,0,0) \qquad (x,y,z)=(0,1,0) \qquad (x,y,z)=(1,1,0)$$

5. Express in spherical coordinates:

$$x^2-y^2 \qquad \frac{x^2+y^2}{z^2} \qquad 2z^2-x^2-y^2 \qquad \left(x^2+y^2+z^2\right)^{-1/2} \qquad \frac{\partial}{\partial x}\left(x^2+y^2+z^2\right)^{-1/2}$$

6. Find the total mass in the indicated volumes by integrating material density distributions:

(a) $\rho = x^2+y^2+z^2$ — V: sphere of radius a with the centre at the origin

(b) $\rho = \dfrac{\sin^2(\theta)\cos^2(\varphi)}{r}$ — V: sphere of radius a with the centre at the origin

(c) $\rho = r^3 e^{-r}$ — V: all space

7. The wavefunction of the 2s orbital of the hydrogen atom is

$$\psi_{2s}(r,\theta,\varphi)=\frac{1}{4\sqrt{2\pi a_0^3}}\left(2-\frac{r}{a_0}\right)e^{-r/2a_0}$$

where a_0 is a fundamental constant. Show that the integral of $\left|\psi_{2s}\right|^2$ over all space is unity.