Problem 1

```
FNorm[f_] := Sqrt[∫_{-1}^{1} Conjugate[f] * f ⅆx];
FScal[f_, g_] := ∫_{-1}^{1} Conjugate[f] * g ⅆx;
```

```
FNorm[x]
```

$$\sqrt{\frac{2}{3}}$$

```
FNorm[2^x]
```

$$\sqrt{\frac{15}{\text{Log}[256]}}$$

```
FNorm[Cos[π x/2]]
```

1

```
FScal[1, x]
```

0

```
FScal[x, 2 x^2 - 1]
```

0

```
FScal[2 x^2 - 1, 4 x^3 - 3 x]
```

0

Problem 2

```
FNorm[f_] := Sqrt[∫_{-π}^{π} Conjugate[f] * f ⅆx];
```

```
A = Assuming[n ∈ Reals, Refine[FNorm[n]]]; Solve[{A == 1, n > 0}, n]
```

$$\left\{\left\{n \to \frac{1}{\sqrt{2\pi}}\right\}\right\}$$

```
A = Assuming[n ∈ Reals, Refine[FNorm[n Sin[x]]]]; Solve[{A == 1, n > 0}, n]
```

$$\left\{\left\{n \to \frac{1}{\sqrt{\pi}}\right\}\right\}$$

```
A = Assuming[n ∈ Reals, Refine[FNorm[Cos[2 x] / n]]]; Solve[{A == 1, n > 0}, n]
```

$$\left\{\left\{n \to \sqrt{\pi}\right\}\right\}$$

```
A = Assuming[n ∈ Reals, Refine[FNorm[n Sin[x] ^2]]]; Solve[{A == 1, n > 0}, n]
```

$$\left\{\left\{n \to \frac{2}{\sqrt{3\pi}}\right\}\right\}$$

```
A = Assuming[n ∈ Reals, Refine[FNorm[n (ⅈ - x / π)]]]; Solve[{A == 1, n > 0}, n]
```

$$\left\{\left\{n \to \frac{1}{2}\sqrt{\frac{3}{2\pi}}\right\}\right\}$$

```
A = Assuming[n ∈ Reals, Refine[FNorm[n Exp[-4 ⅈ x]]]];
Solve[{A == 1, n > 0}, n]
```

$$\left\{\left\{n \to \frac{1}{\sqrt{2\pi}}\right\}\right\}$$

Problem 3

$$\frac{1}{\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left(\mathrm{Exp}\left[-x^2 - y^2\right]\right) dx\, dy$$

1

$$\frac{2}{\pi}\int_{-\infty}^{\infty}\left(\frac{1}{x^2+1}\right)^2 dx$$

1

Problem 4

Wronskian below is an approximate criterion for linear independence.

```
(Wronskian[{x, 1 + x, 1 - x}, x] /. {x → 1}) ≠ 0
```

False

```
(Wronskian[{Cos[x], Cos[2 x], Cos[3 x]}, x] /. {x → 1}) ≠ 0
```

True

```
(Wronskian[{0, x Log[x], ArcTan[x], Exp[x]}, x] /. {x → 1}) ≠ 0
```

False

```
(Wronskian[{Log[x], Log[x^2], Log[x^3]}, x] /. {x → 1}) ≠ 0
```

False

Problem 5

The expansion coefficients for the three functions are in the curly brackets.

$$\mathrm{FScal}[f\_, g\_] := \int_{-\pi}^{\pi} \mathrm{Conjugate}[f] * g\, dx;$$

$$\mathrm{Thread}\left[\mathrm{FScal}\left[\left\{\mathrm{Exp}[-i\, x] \big/ \sqrt{2\pi},\ 1 \big/ \sqrt{2\pi},\ \mathrm{Exp}[i\, x] \big/ \sqrt{2\pi}\right\}, \mathrm{Cos}[x]\right]\right]$$

$$\left\{\sqrt{\frac{\pi}{2}},\ 0,\ \sqrt{\frac{\pi}{2}}\right\}$$

```
Thread[FScal[{Exp[-ⅈ x]/Sqrt[2 π], 1/Sqrt[2 π], Exp[ⅈ x]/Sqrt[2 π]}, Sin[x]]]
```

$\left\{i\sqrt{\frac{\pi}{2}},\ 0,\ -i\sqrt{\frac{\pi}{2}}\right\}$

```
Thread[FScal[{Exp[-ⅈ x]/Sqrt[2 π], 1/Sqrt[2 π], Exp[ⅈ x]/Sqrt[2 π]}, 1]]
```

$\left\{0,\ \sqrt{2\pi},\ 0\right\}$

Problem 6

$$\text{Assuming}\left[n \in \text{Integers} \,\&\&\, k \in \text{Integers} \,\&\&\, n \neq k,\ \frac{1}{2\pi}\int_{-\pi}^{\pi} \text{Exp}[-i\, n\, x]\ \partial_{x,x}\text{Exp}[i\, k\, x]\, dx\right]$$

0

$$\text{Assuming}\left[n \in \text{Integers} \,\&\&\, k \in \text{Integers} \,\&\&\, n == k,\ \frac{1}{2\pi}\int_{-\pi}^{\pi} \text{Exp}[-i\, n\, x]\ \partial_{x,x}\text{Exp}[i\, k\, x]\, dx\right]$$

$-n^2$